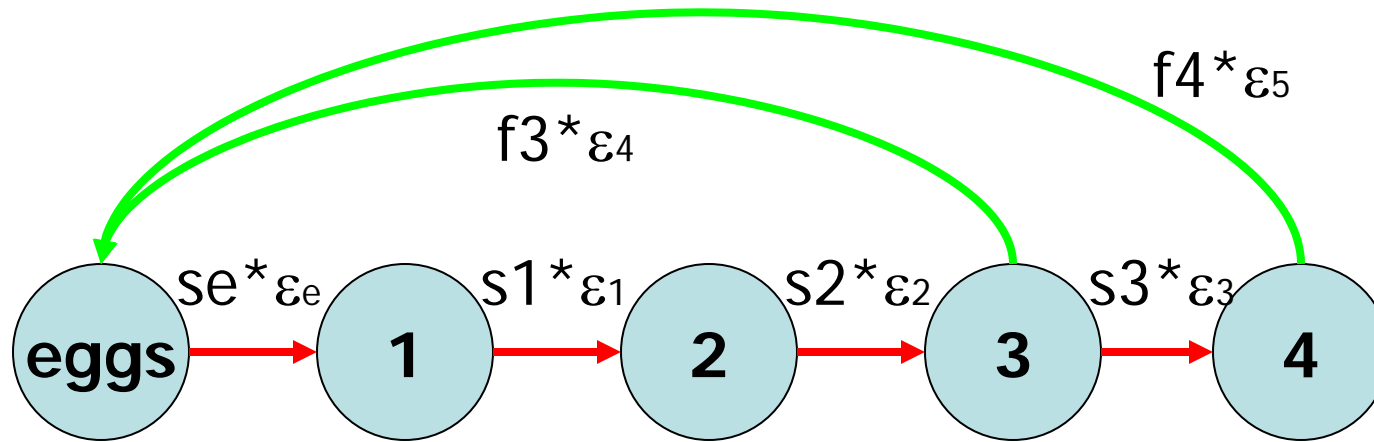


diffusion approximation
approaches for metapopulation
viability analysis

'stochastic approximation' approach: example from single population



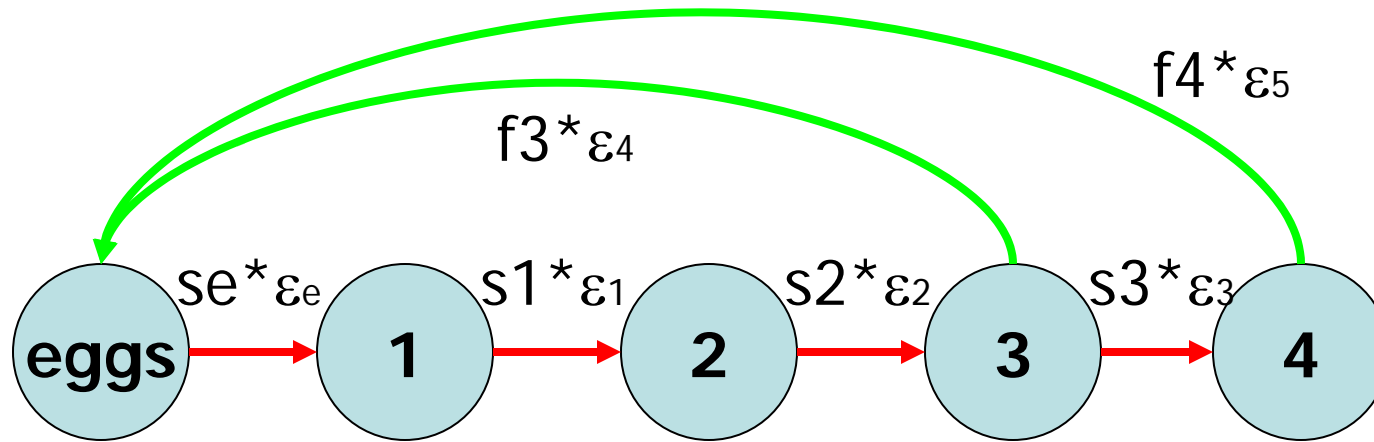
= **Asymptotic stochastic model**

$$\ln N_{t+\tau} = \ln N_t + \mu\tau + \varepsilon, \quad \varepsilon \sim N(0, \sigma \sqrt{\tau})$$

≈ **Markov or diffusion approximation**

$$\ln N_{t+1} = \ln N_t + \mu + \varepsilon, \quad \varepsilon \sim N(0, \sigma)$$

corrupted diffusion model



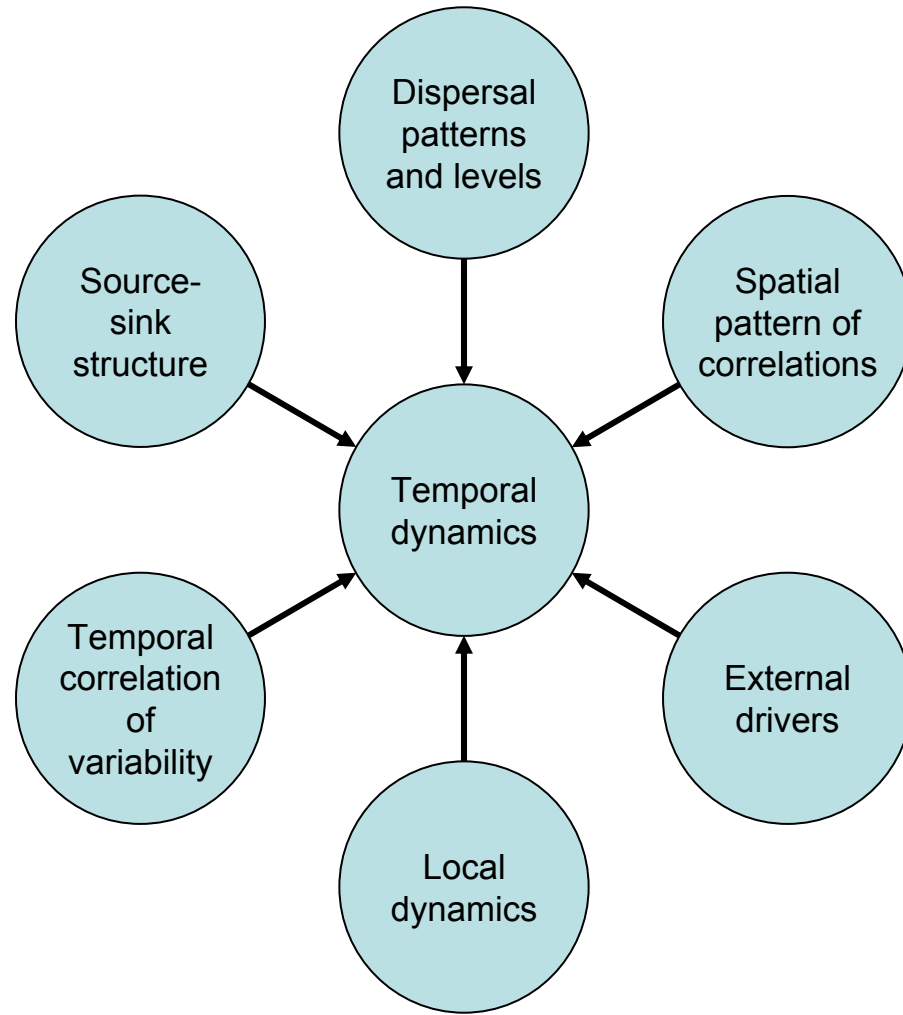
+ Various types of observation and other extraneous variance

≈ state-space model

$$\ln N_{t+1} = \ln N_t + \mu + \varepsilon, \quad \varepsilon \sim N(0, \sigma)$$

$$\ln Y_{t+1} = \ln Y_{t+1} + \varepsilon_s, \quad \varepsilon_s \sim f(v, \sigma_s)$$

lots of things affect metapopulation dynamics



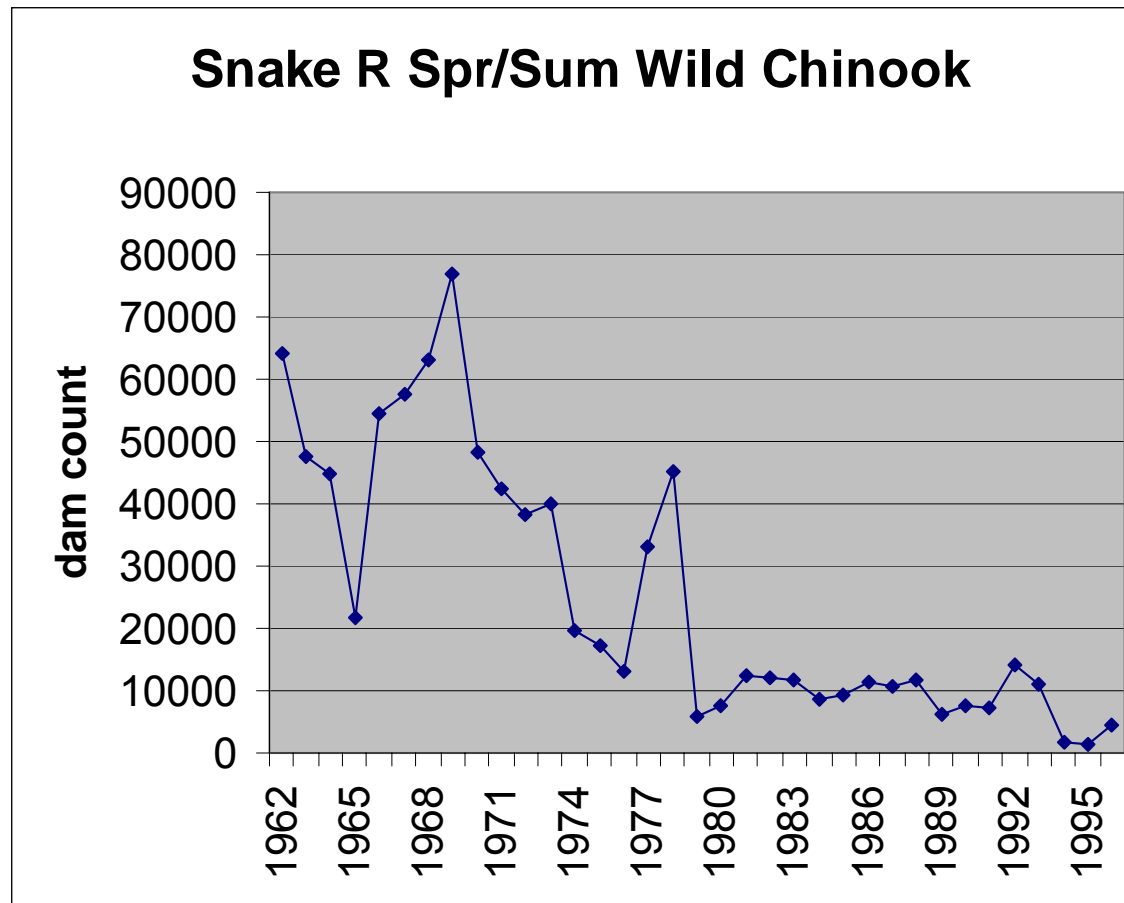
IUCN Red List Criteria

- Criteria A2: “A reduction of at least xx%, projected or suspected to be met within the next xx years....”
- Criteria C1: “Population estimated to number less than xx and an estimated continuing decline of at least xx% within xx years....”
- Criteria E: “Quantitative analysis showing the probability of extinction in the wild is at least xx% within xx years...”

diffusion approximations for metapopulations

- A theoretical framework for 'diffusion approximations' for metapopulations
- Application to declining metapopulations
- Example using chinook salmon
- Some conjectures regarding management implications

focus on declining metapopulations



local internal dynamics

$$N_i(t) = N_i(t-1) \underline{e^{z_i(t-1)}} \quad \text{stochastic population growth}$$

$$\underline{-d_i(t-1)N_i(t-1)e^{z_i(t-1)}} \quad \text{dispersal out}$$

$$+ \sum_{j \neq i} \underline{\alpha_{ji}(t-1)d_j(t-1)} \underline{N_j(t-1)e^{z_j(t-1)}} \quad \text{dispersal in}$$

- Any spatial pattern of dispersal (any kernel)
- Any temporally varying distributions of dispersal rates as long as the distributions are stationary
- Any spatial pattern of correlation in growth rates among sites
- z's need not be normally distributed

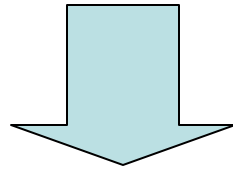
Matrix representation of stochastic metapopulation

$$\begin{bmatrix} N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \\ \dots \\ N_k(t+1) \end{bmatrix} = \mathbf{A}(t) \times \begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \\ \dots \\ N_k(t) \end{bmatrix}$$

stochastic matrix

$$\mathbf{A}(t) = \begin{bmatrix} (1-d_1)e^{z_1} & \alpha_{21}d_2e^{z_2} & \alpha_{31}d_3e^{z_3} & \dots & \alpha_{k1}d_ke^{z_k} \\ \alpha_{12}d_1e^{z_1} & (1-d_2)e^{z_2} & \alpha_{32}d_3e^{z_3} & \dots & \alpha_{k2}d_ke^{z_k} \\ \alpha_{13}d_1e^{z_1} & \alpha_{23}d_2e^{z_2} & (1-d_3)e^{z_3} & \dots & \alpha_{k3}d_ke^{z_k} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{1k}d_1e^{z_1} & \alpha_{2k}d_2e^{z_2} & \alpha_{3k}d_3e^{z_3} & \dots & (1-d_k)e^{z_k} \end{bmatrix}$$

$$\begin{bmatrix} N_1(t + \tau) \\ N_2(t + \tau) \\ N_3(t + \tau) \\ \dots \\ N_k(t + \tau) \end{bmatrix} = \underbrace{\mathbf{A}(t + \tau - 1) \times \mathbf{A}(t + \tau - 2) \times \dots \times \mathbf{A}(t + 1) \times \mathbf{A}(t)}_{\text{Product of random matrices}} \times \begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \\ \dots \\ N_k(t) \end{bmatrix}$$

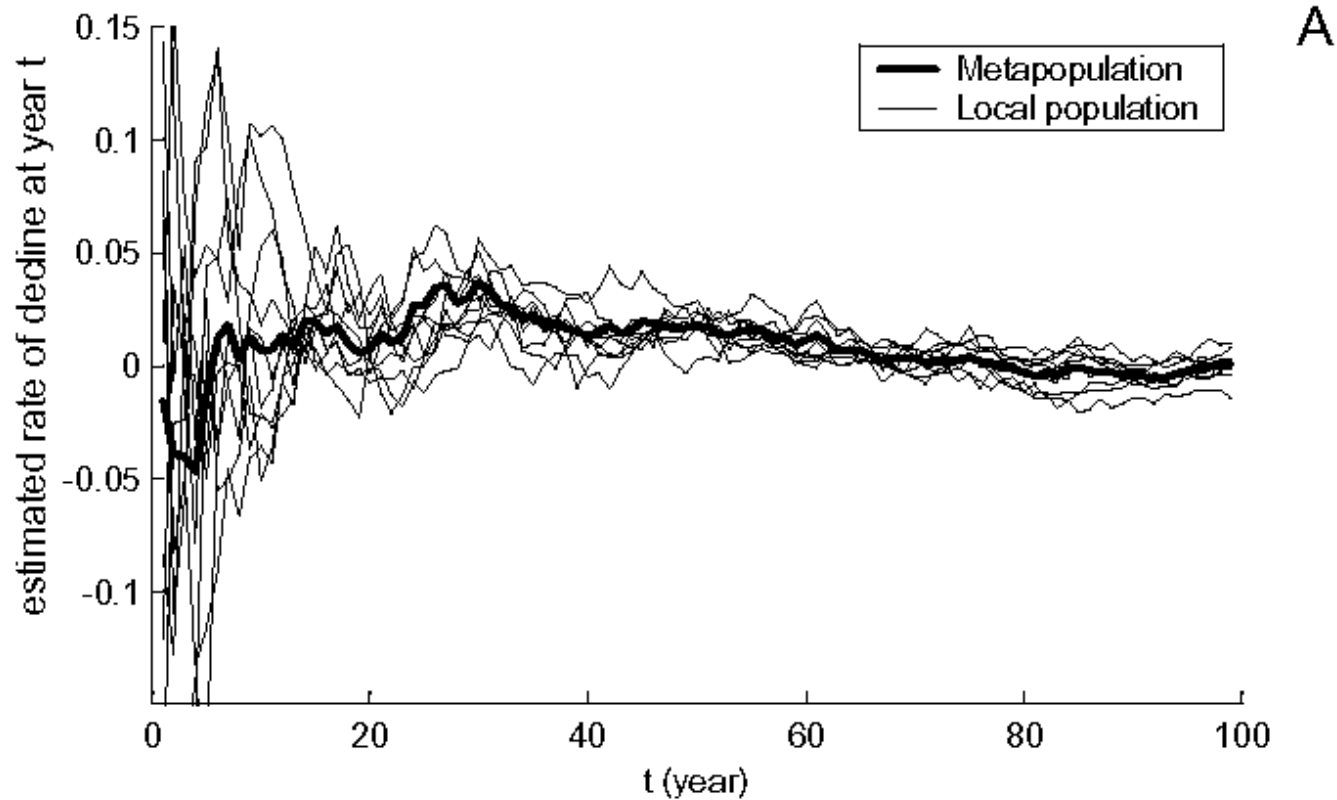


$$\ln M(t + \tau) \xrightarrow{t \rightarrow \infty} \ln M(t) + \tau \mu_m + \varepsilon$$

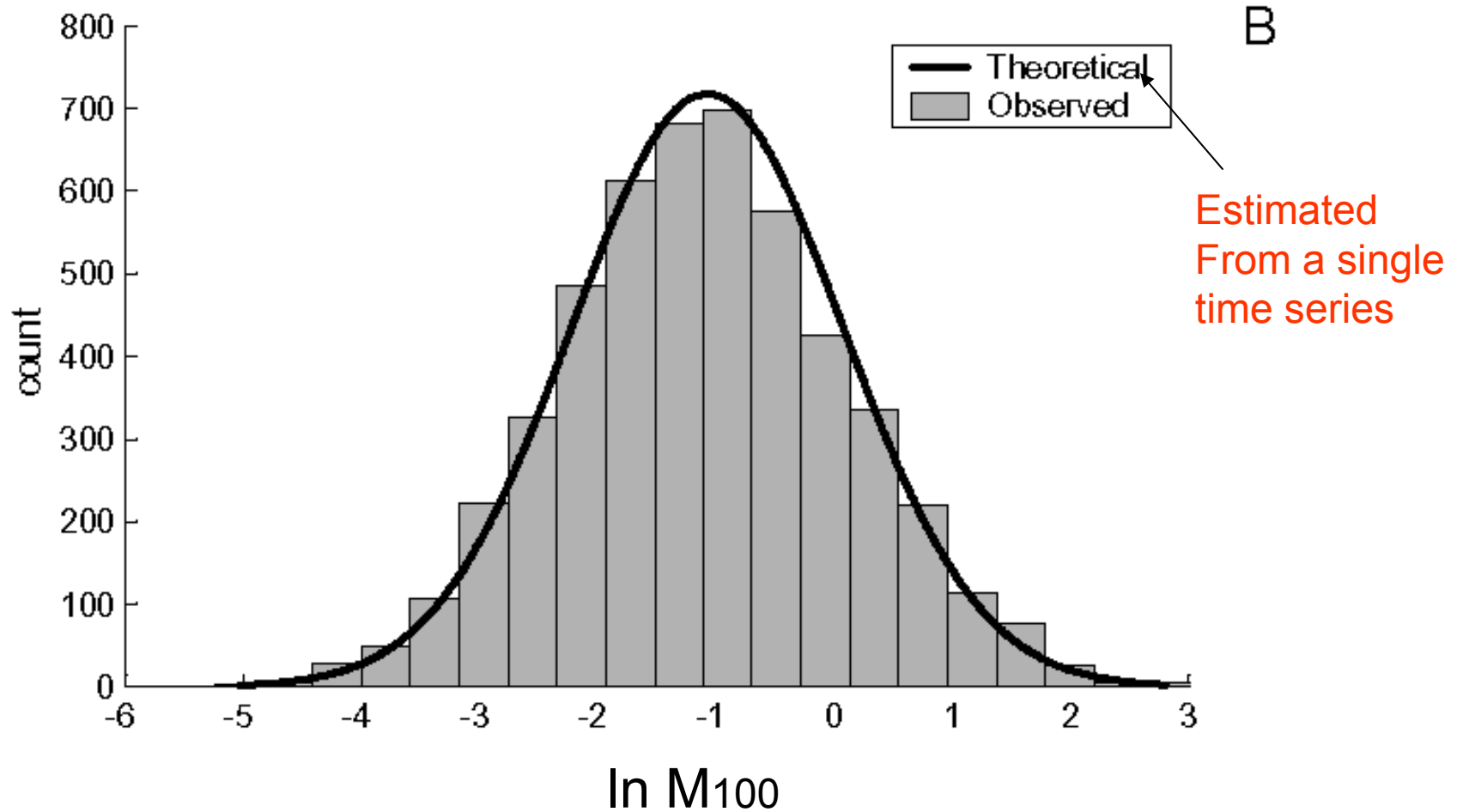
$$\ln N_i(t + \tau) \xrightarrow{t \rightarrow \infty} \ln N_i(t) + \tau \mu_m + \varepsilon$$

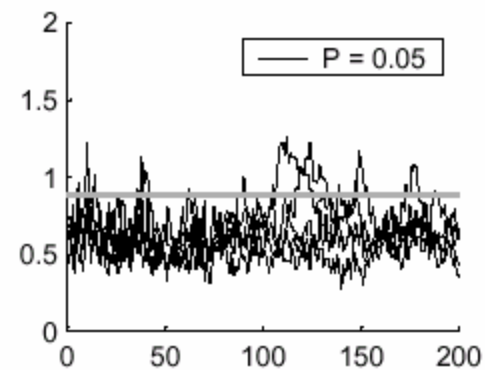
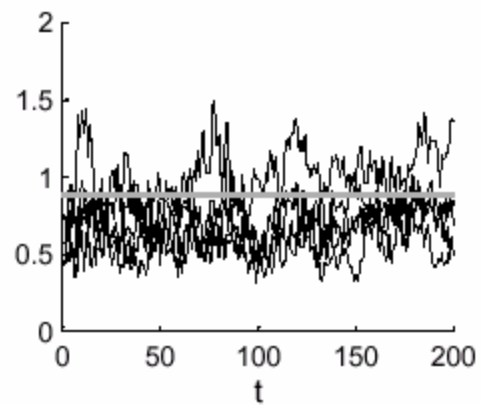
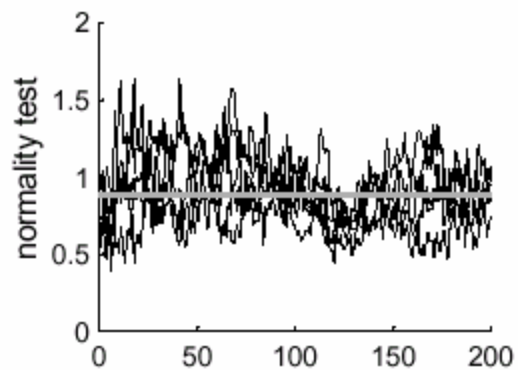
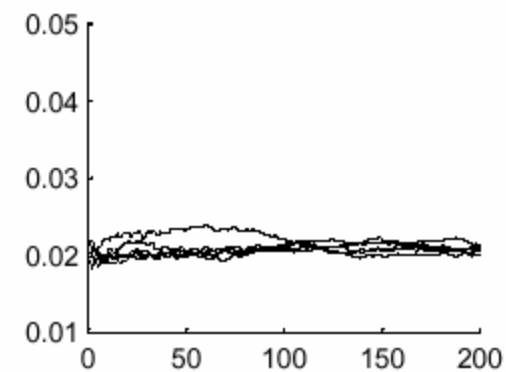
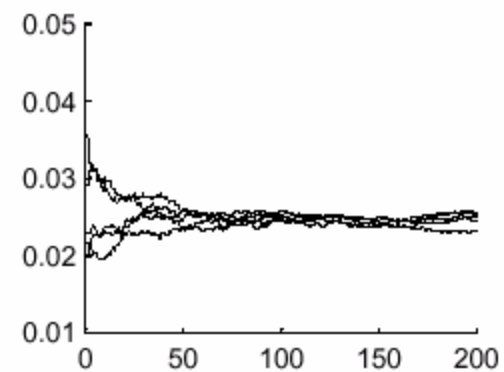
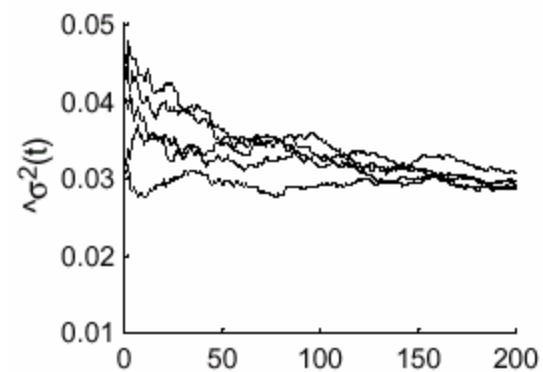
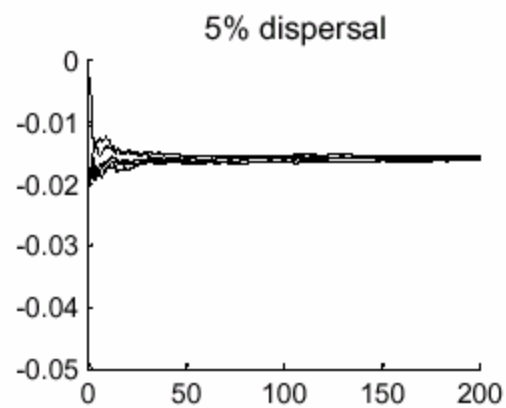
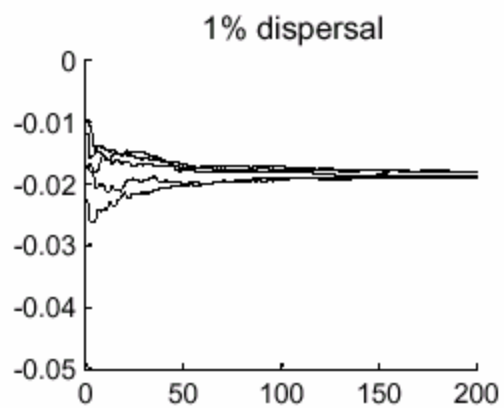
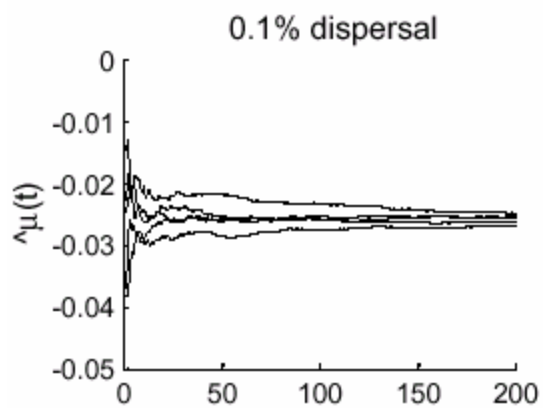
$$\varepsilon \sim \text{normal}(0, \sigma \sqrt{\tau})$$

simulated example of long-term dynamics

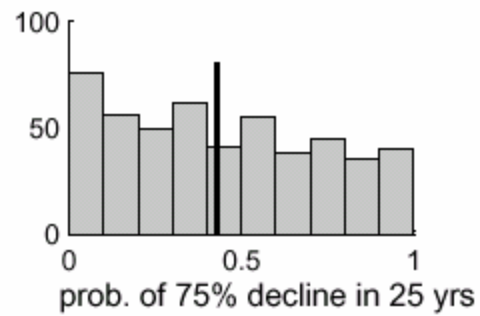
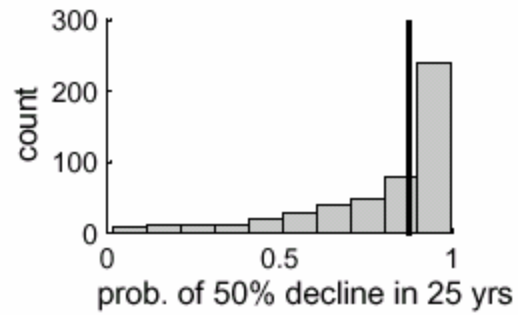
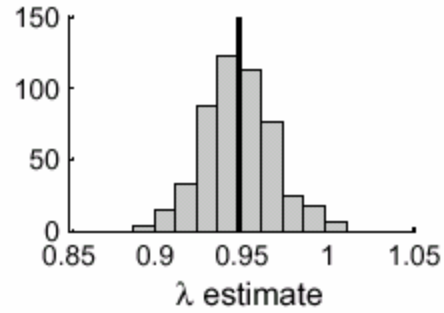


observed vs theoretical distribution of metapopulation size

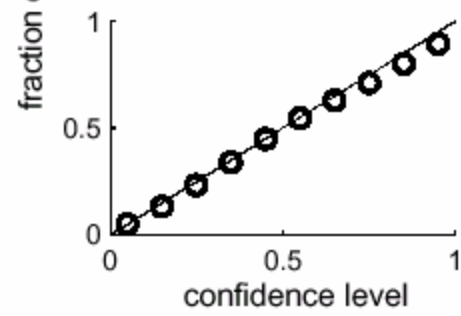
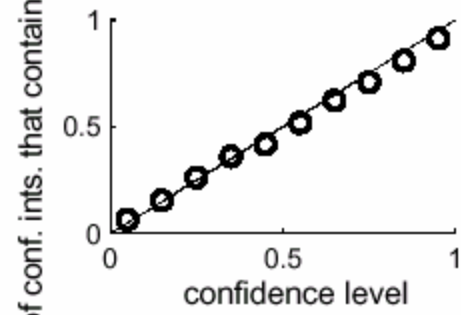
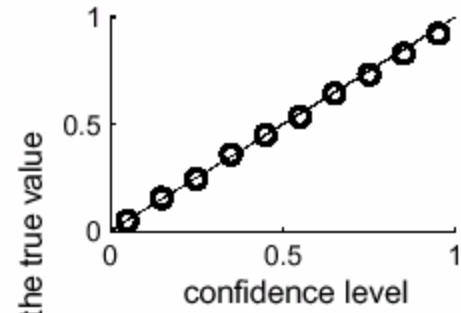




— true value
 ■ distribution of estimates



● actual relationship
 — correct relationship



from theory to application

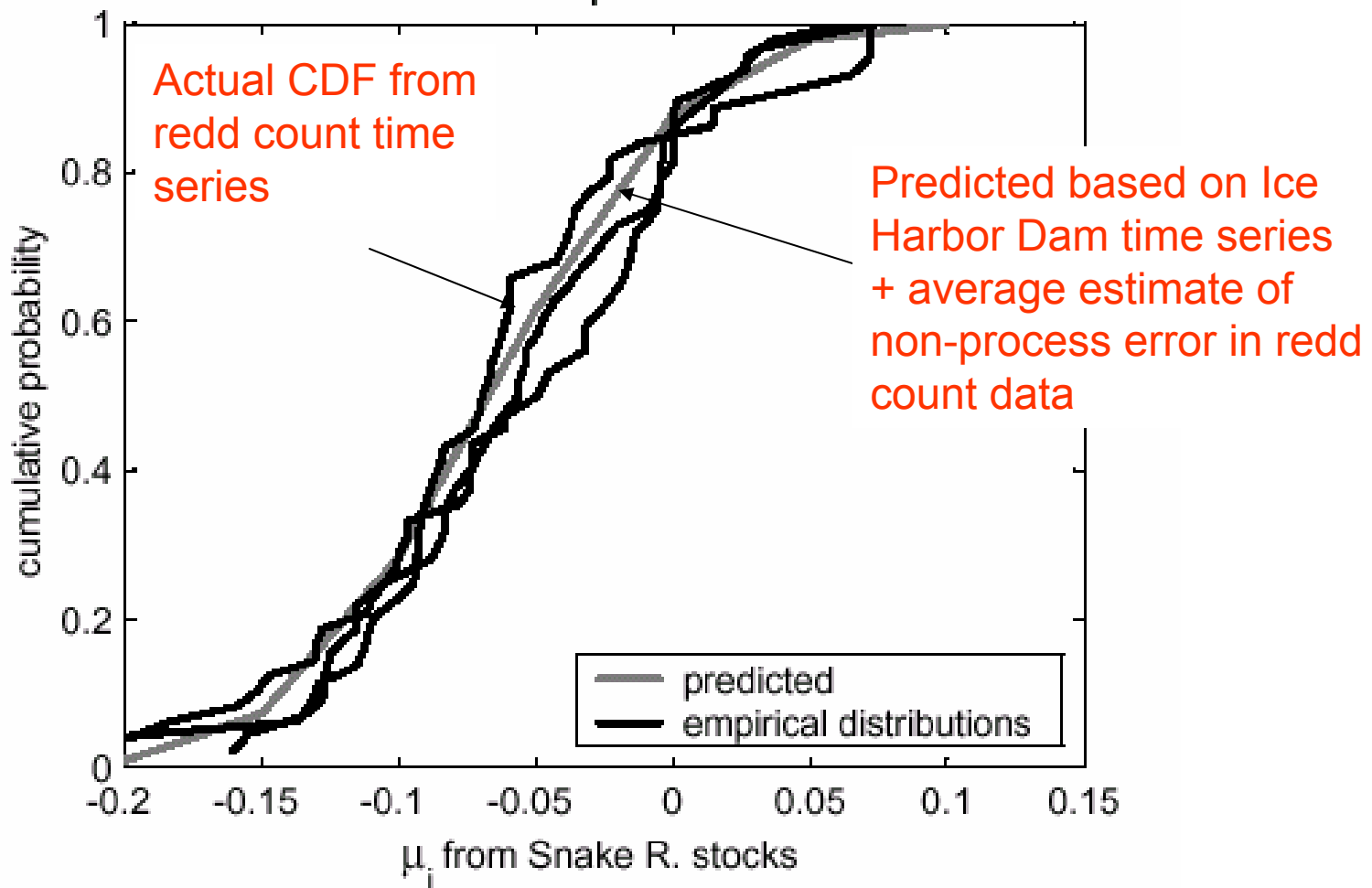
- Develop of approximation: $\ln M(t)/M(0) = \text{normal}(t\mu, t\sigma)$
- Develop of statistical methods estimating parameters: μ and σ
- Cross-validation using data
- Use of approximations for PVA purposes



Snake R Spr/Sum Chinook Salmon

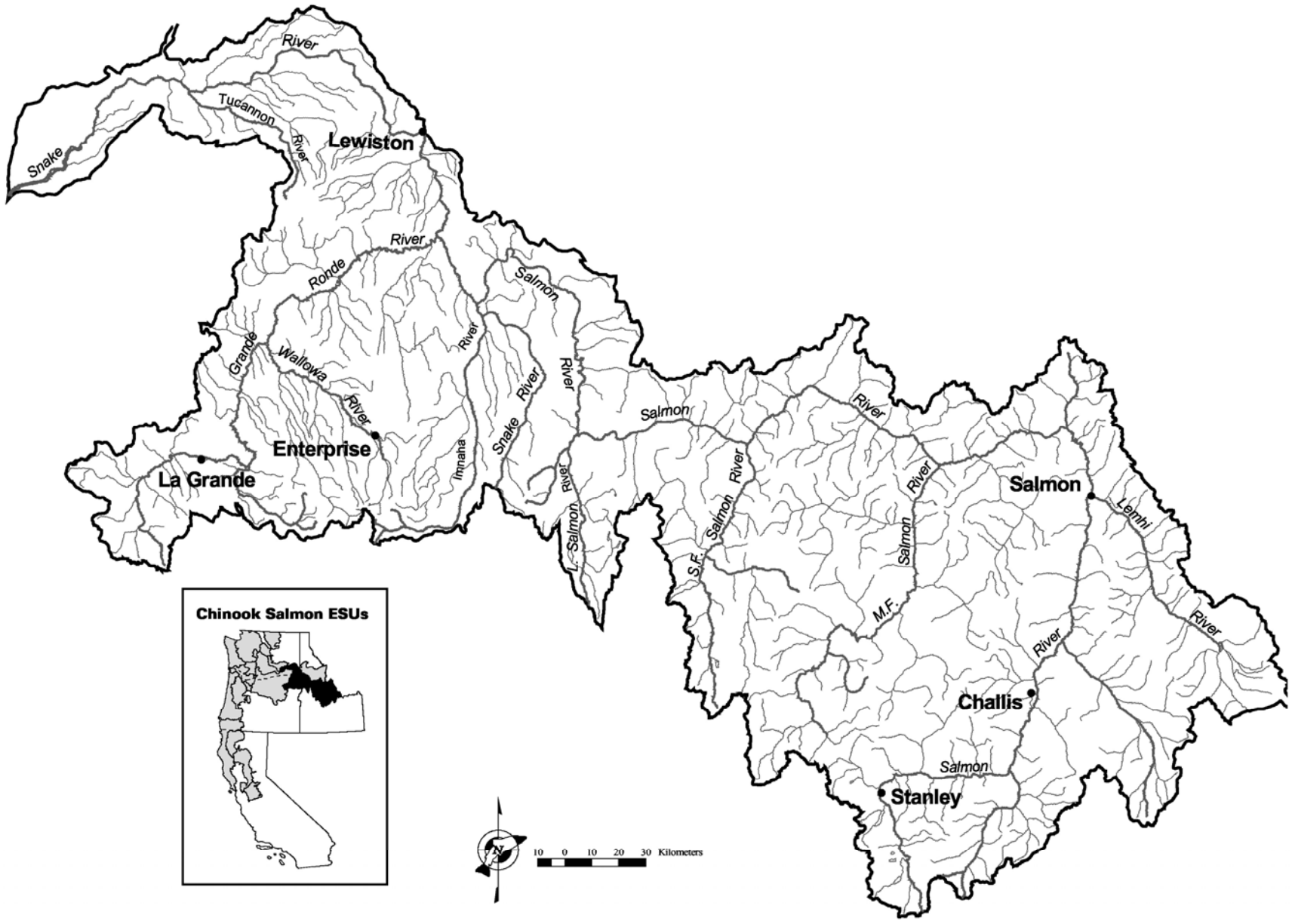


Snake River spr/sum chinook



management conjectures





Observed subpop rates of decline do not reflect local conditions

- *At equilibrium*, observed differences in local trends occur by chance
- Actual local conditions (plus connectivity) are reflected in local density



NOAA Technical Memorandum NMFS F/NWC-202

Status Review
for
Lower Columbia River Coho Salmon

by
Orlay W. Johnson, Thomas A. Flagg,
Desmond J. Maynard, George B. Milner,
and F. William Waknitz

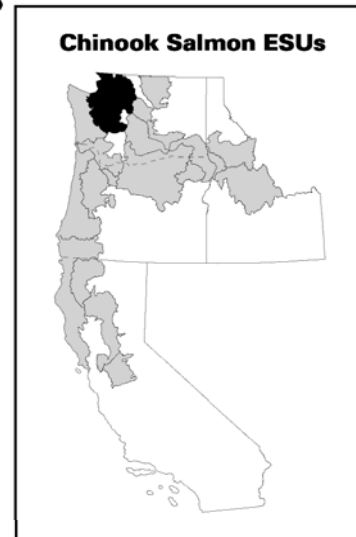
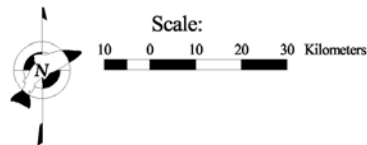
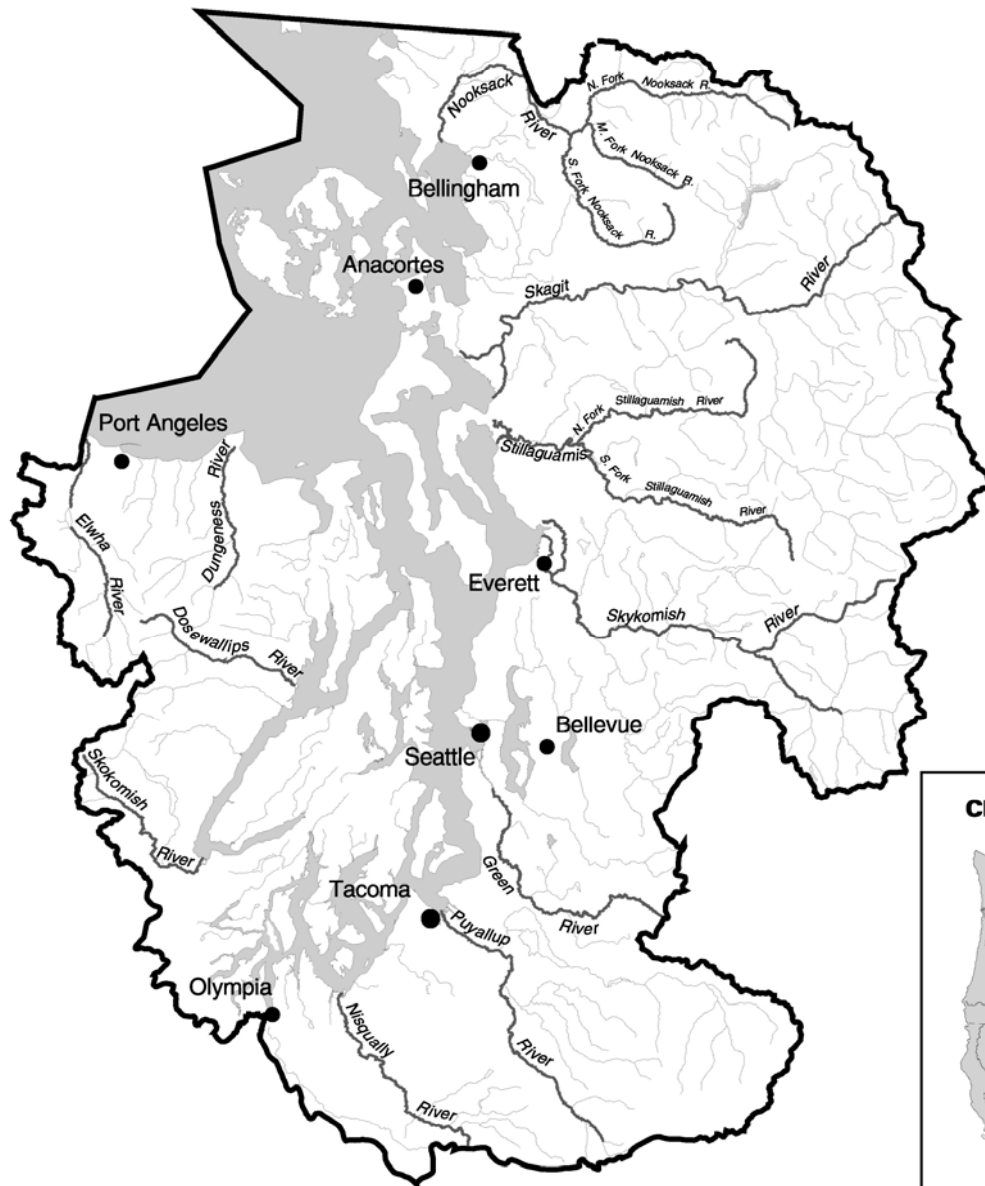
June 1991

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Marine Fisheries Service

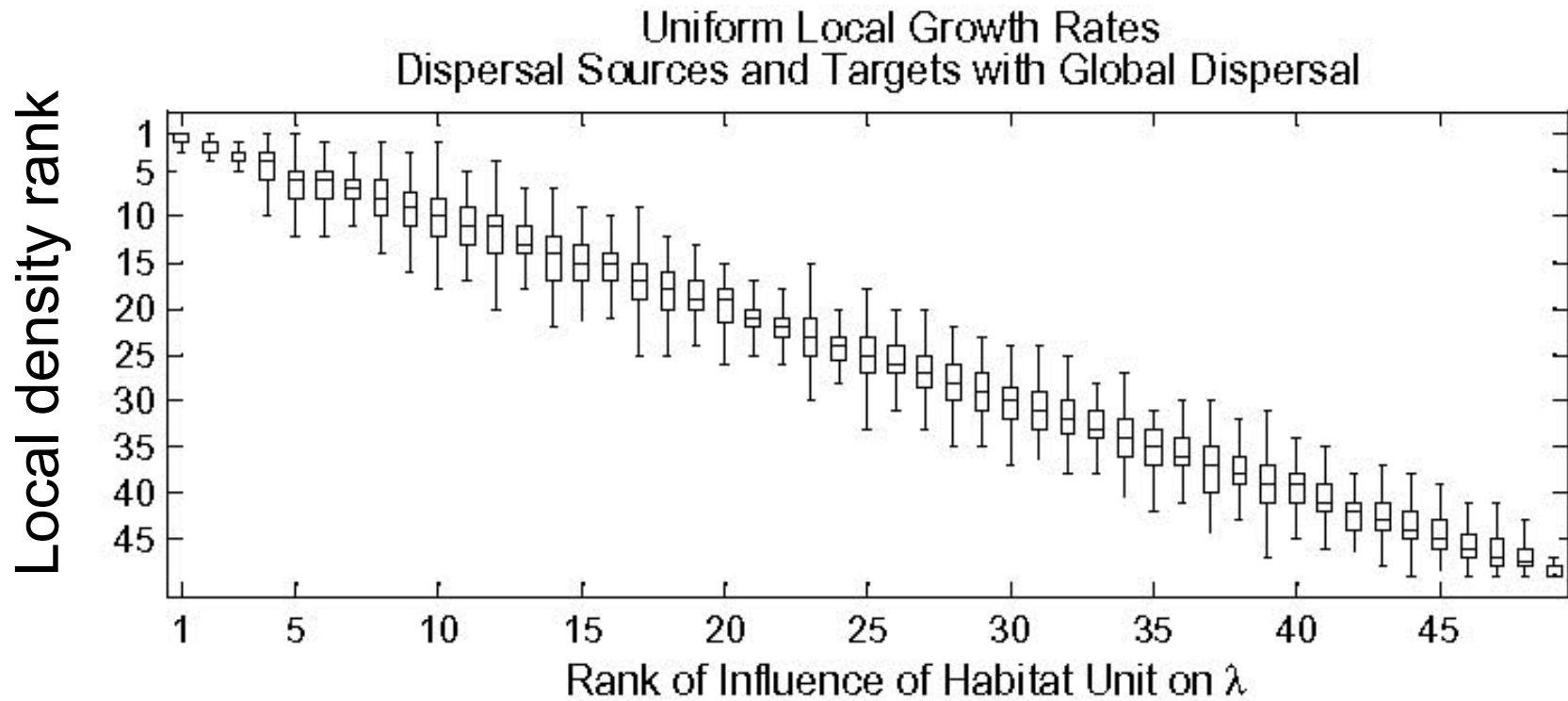
This TM series is used for documentation and timely communication of preliminary results, interim reports, or special purpose information, and has not received complete formal review, editorial control, or detailed editing.

- This work has implications for how trends of individual stocks in a metapopulation are interpreted.
- How do you account for non-equilibrium dynamics?
- Transients?

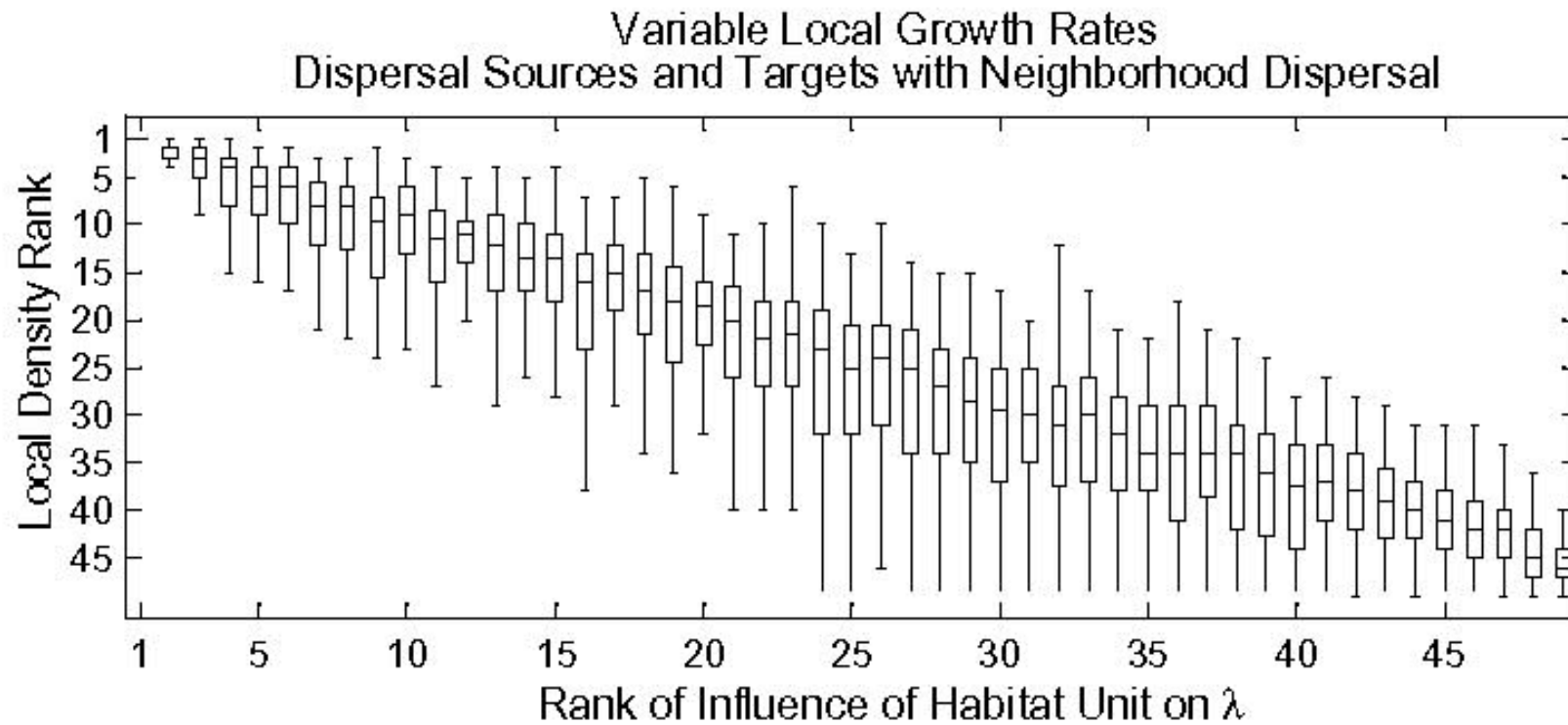
Puget Sound Chinook



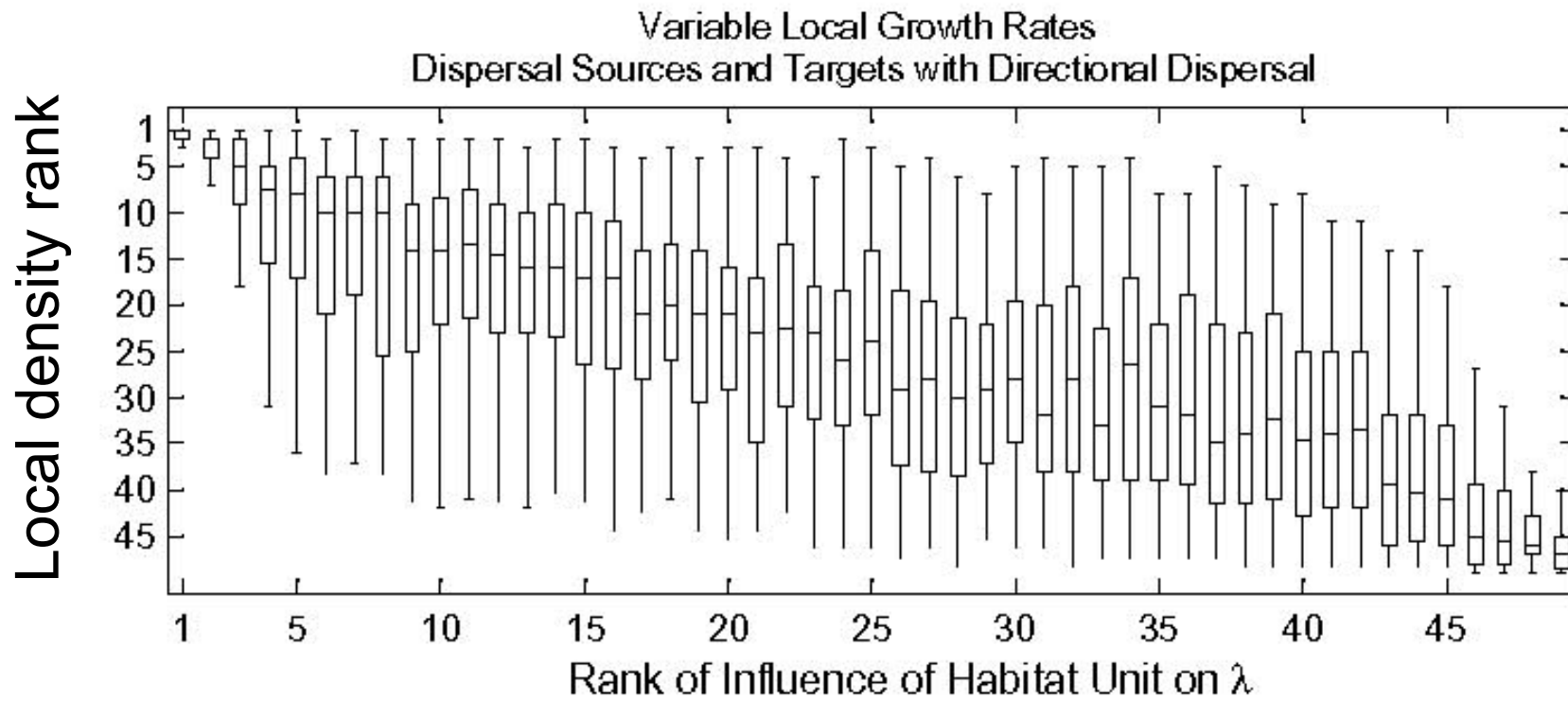
“bang-for-effort”



“bang-for-effort”



“bang-for-effort”





- Somewhat intuitive implications for distribution of habitat protection efforts → protect pristine areas with high population densities
- Counter-intuitive in-stream harvest closures → allow harvest in low density-streams and close harvest in high-density streams

