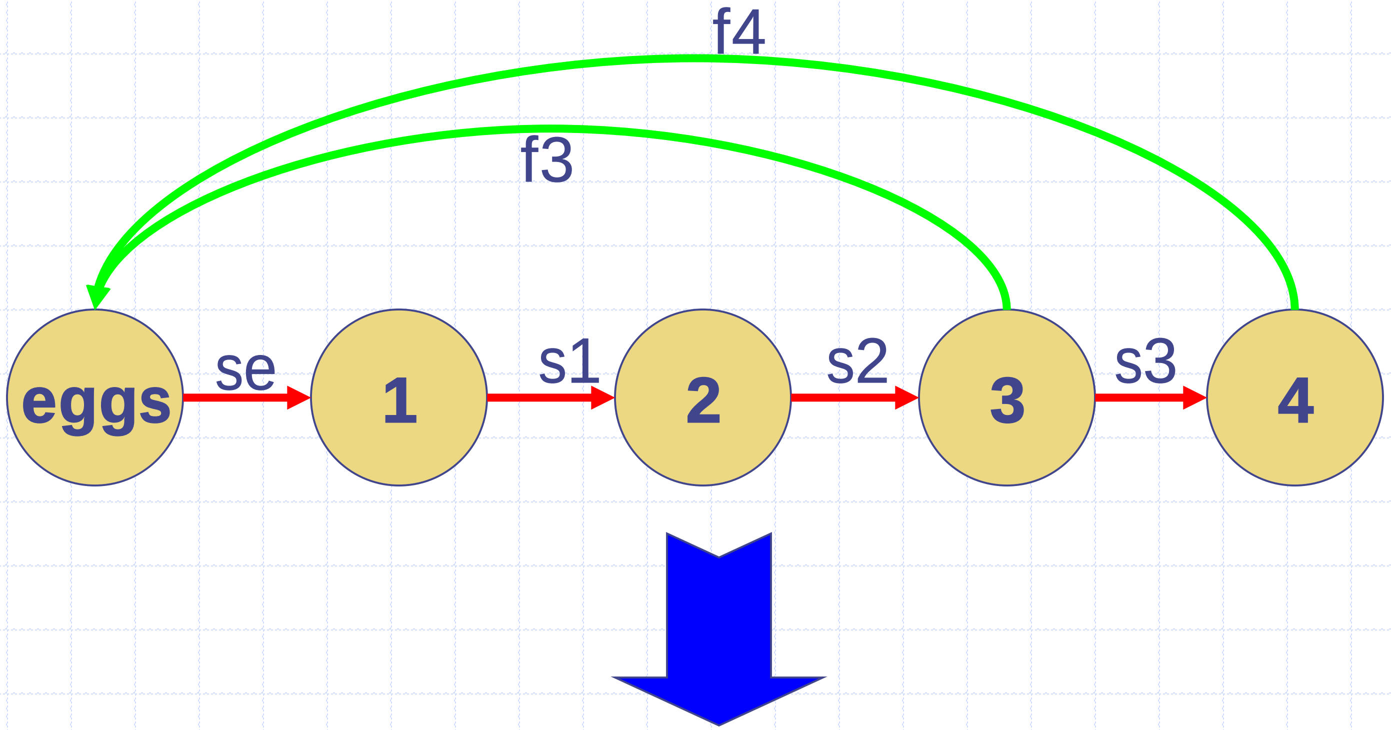
Estimating Risks using Diffusion Approximations

- Overview of DAs for age-structured populations
- The joys of error-ridden data
- Cross-validation
  - With models
  - With data
- Applications

What and Why?

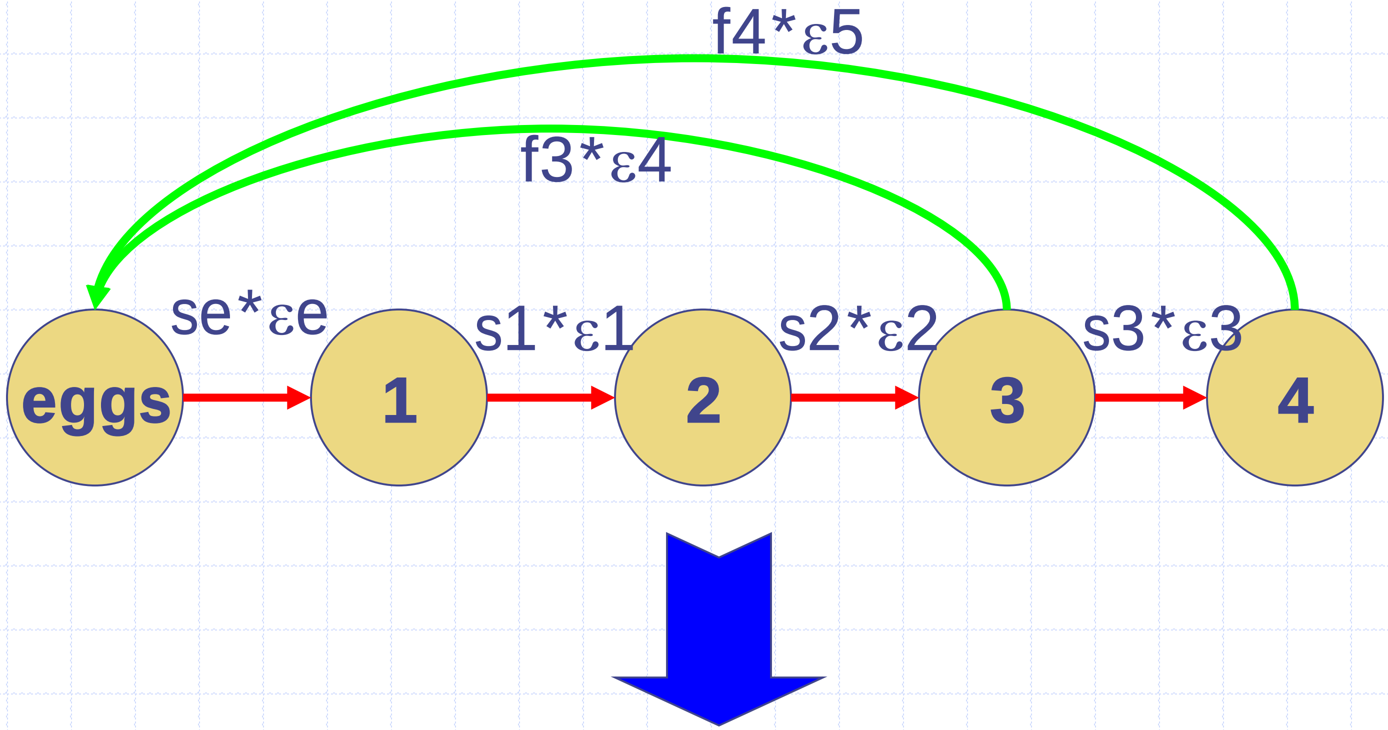
- Dennis model: Count-based population viability analysis based on an approximation of an age-structured stochastic population with a simple stochastic model.
- Dennis-Holmes method: a method estimating the parameters of that model from highly problematic data

Foundations: age-structured models



$$N_{t+\tau}=N_t*\exp(\mu\tau)$$

Foundations: add environmental variability



$N_{t+\tau}=N_t*\exp(\mu\tau+\varepsilon)$ where

$\varepsilon \sim N(0,\sigma\ \mathrm{sqrt}(\tau))$

Parameters

$N_{t+\tau}=N_t*\exp(\mu\tau+\varepsilon)$ where $\varepsilon \sim N(0,\sigma\, \mathrm{sqrt}(\tau))$

Parameter that governs the median rate of decline.

"Process error": parameter that governs how fast the variability in $N_{t+\tau}$ increases

What's so nice about:

$N_{t+\tau} = N_t * \exp(\mu\tau + \varepsilon)$ where $\varepsilon \sim N(0, \sigma \, \mathrm{sqrt}(\tau))$

- This process can be approximated by a diffusion equation which we can then use to calculate lots of useful metrics of its stochastic behavior such as
  - Probability of extinction or quasi-extinction
  - Mean time to extinction
  - Mean and median rate of growth/decline

Parameter estimation for nice data: Dennis method

Whooping cranes 1938-1947

count	18	22	26	16	19	21	18	22	25	31
Ln(Nt+1/Nt)	.20	.16	-.48	.17	.10	-.15	.20	.13	.21	

μ estimate = .038 σ^2 estimate = .05

Median rate of growth = exp(.038) = 1.038

Prob that cranes halve in 20 years = 7 %

Caveat: point estimates are a poor way to present results from DAs

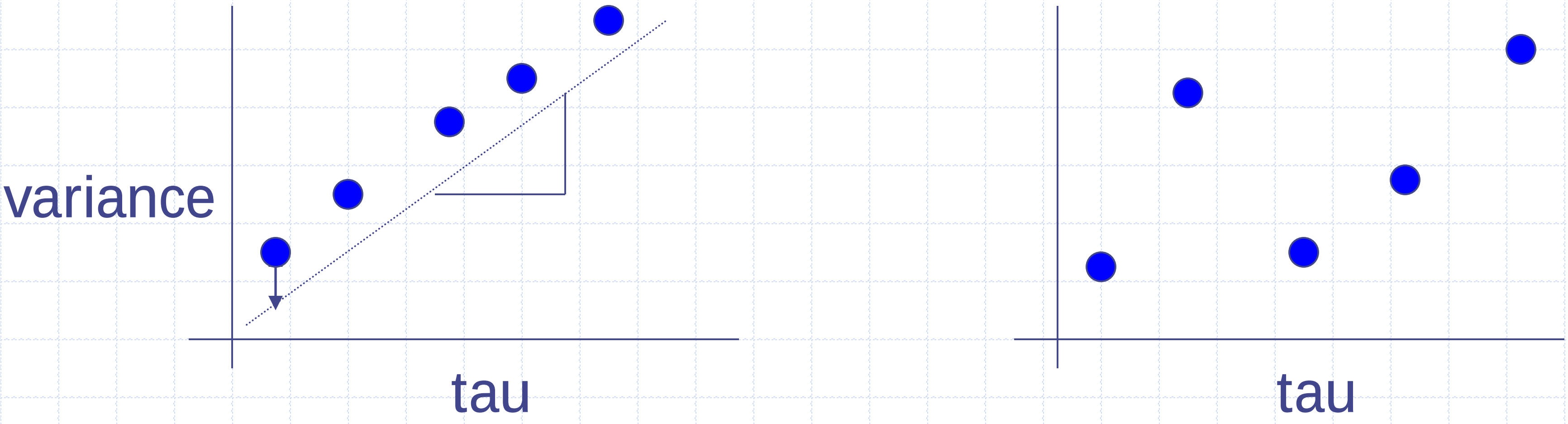
Problematic data ☹

- High sampling error
- Age or stage specific censuses
- Tendency for age-structure fluctuations (boom-bust cycles)
- Non-equilibrium

Severe over-estimates of σ^2
Severe = 10,000-30,000 % over-estimates for example for salmon

Alternate method (Holmes 2001)

- $N_{t+\tau} = N_t{*}\exp(\mu\tau+\varepsilon)$, $O_t = N_t{*}\exp(\varepsilon_s)$,
- $\ln(O_{t+\tau}/O_t) = \mu\tau + \varepsilon_s + \varepsilon\tau$
- Variance of $\ln(O_{t+\tau}/O) = \sigma^2{}_s + \sigma^2\tau$



The trick: running sums

- $R_t = N_t + N_{t+1} + N_{t+2} + N_{t+3}$ retains statistical properties of Nt due to lognormality of N_t and correlation between N_t and N_{t+1}
- R_t filters out a lot of the non-process error so that you can see the $(c + \sigma^2\tau)$ relationship
- μ estimate from R_t is also more stable

But there's no free lunch

- Significant reduction in bias of σ^2 estimate comes with an increase in the variance of the σ^2 estimate
- Non-process error inflates the variance in the μ estimate

Leslie matrix model

$$\mathbf{N}_{t+1} = \mathbf{A} \times \mathbf{N}_t$$

$$A = \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & p\, s_1\, m\, \varepsilon_1 \\ s_2\varepsilon_2 & 0 & 0 & 0 & 0 \\ 0 & (1-b_3)s_3\varepsilon_3 & 0 & 0 & 0 \\ 0 & 0 & (1-b_4)s_4\varepsilon_4 & 0 & 0 \\ 0 & b_3 s_3 \varepsilon_3 & b_4 s_4 \varepsilon_4 & b_5 s_5 \varepsilon_5 & 0 \end{array}\right] \begin{array}{c} \text{egg, age 1} \\ \text{age 2} \\ \text{age 3} \\ \text{age 4} \\ \text{spawners} \end{array}$$

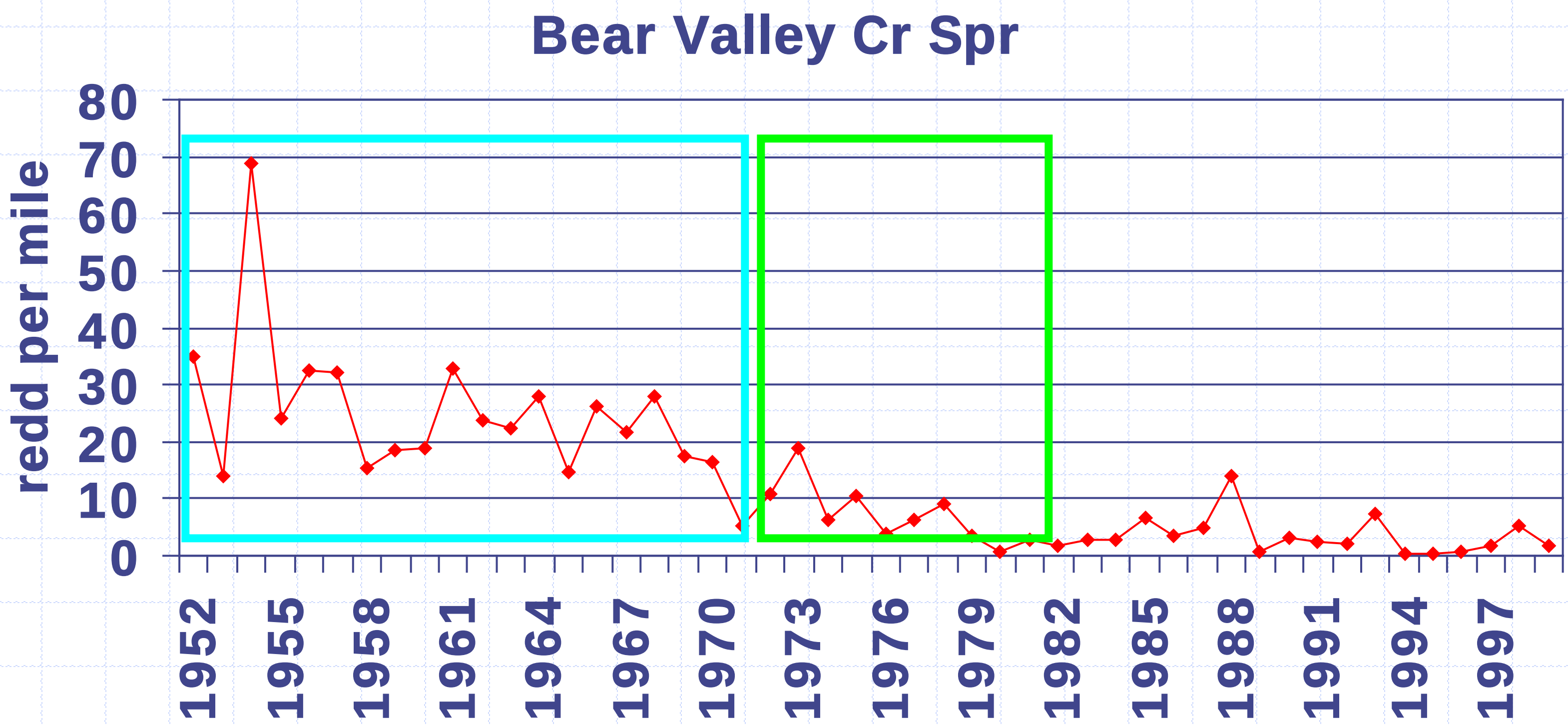
Cross-validation with age-structured models

- Published Leslie matrix models for sea turtles, storm petrels, spr/sum chinook, fall chinook, steelhead
- Plus extreme sampling error
- Plus non-equilibrium age-structure

- Dennis method: 2,500 to 10,000% median errors in σ^2
- Alternate method: 50 to 250% median errors in σ^2

Cross-validation

- **141 chinook and 41 steelhead 30-70 year time series from ESUs in WA, OR, and CA**



Metrics

- Probability of x decline at the end of 10 years
- Probability of hitting thresholds within a 10 year period
- Do observed rates of decline the expected distribution?
- Do the σ^2 estimates fit the expected distributions?