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# A statistical approach to PVA validation: an example using diffusion approximation models

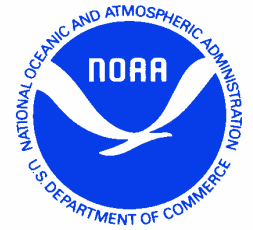
Updated with a few comments after talk

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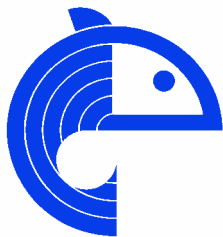


# Overview

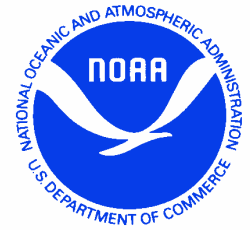
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- u Different validation approaches
- u Example: a cross-validation study of diffusion approximation PVAs
- u Presenting uncertainty.

FOR MORE INFO...



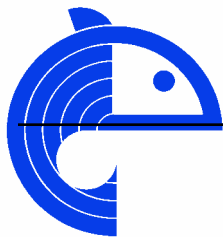
[faculty.washington.edu/eeholmes](http://faculty.washington.edu/eeholmes)



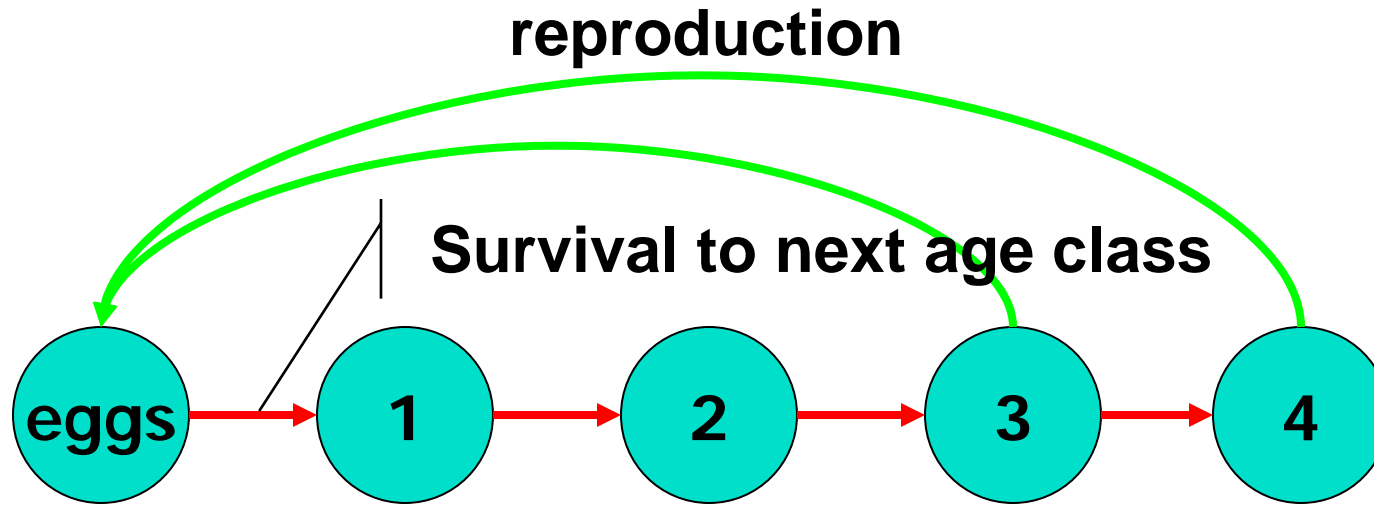
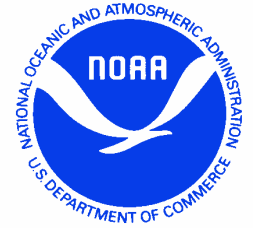
# Methods for testing PVAs

from McCarthy et al. 2002. "Testing the Accuracy of PVA" Cons. Bio.

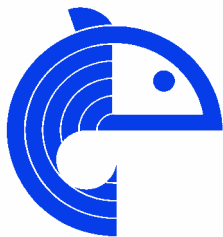
Compare mean or median predictions with observations	Subjective, ignores variability, single trajectories unlikely to be similar to mean
Compare observed vs predicted frequency of events	Only assesses average number or frequency of occurrences within a group, ignores variability
Compare probability distributions of population size or parameters	Assesses both the mean and variability, generally requires transformation of data to a standard variate, lots of data



# Diffusion approximation PVA

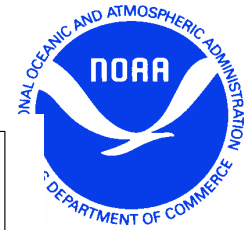


**Let reproduction and survival vary yearly**

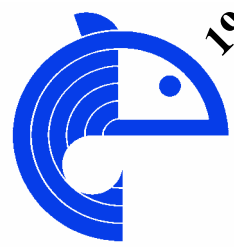
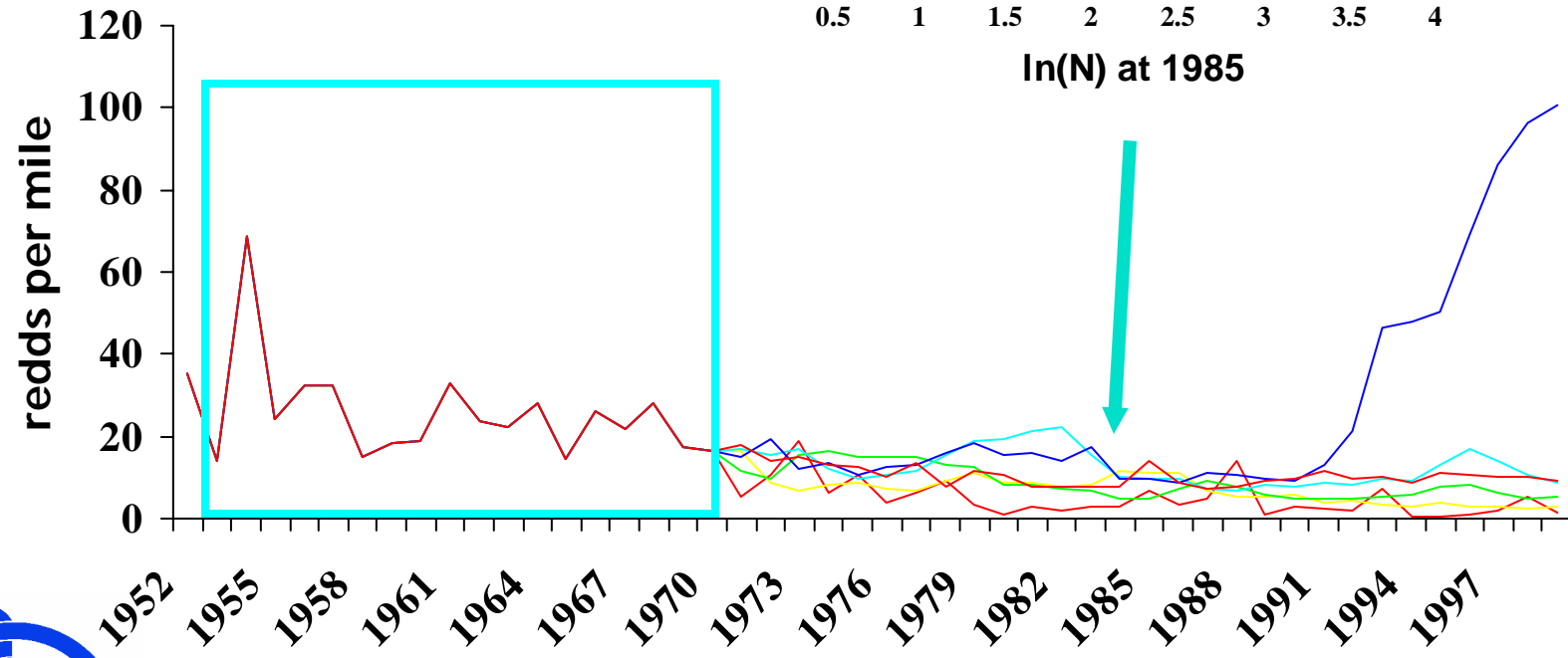
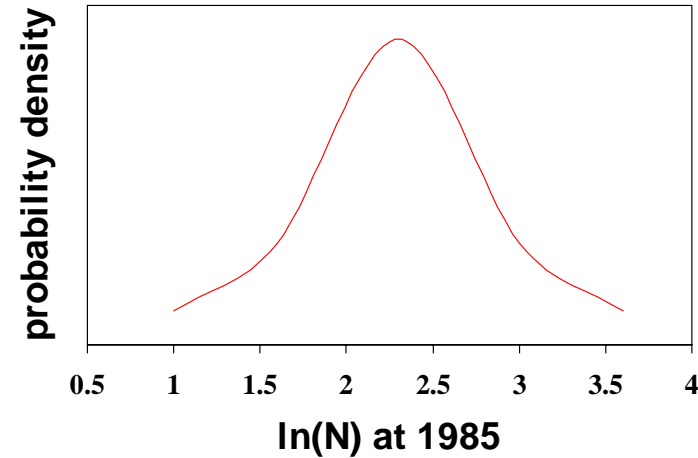


$$N_t = N_0 \exp(\mu t + \varepsilon t)$$

where  $\varepsilon \sim \text{Normal}(0, \sigma)$



# Basic Idea of DA $PV \Delta$



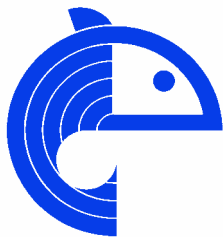
# Parameters of a DA model

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$$N_t = N_0 * \exp(\mu t + \varepsilon t) \text{ where } \varepsilon \sim N(0, \sigma)$$

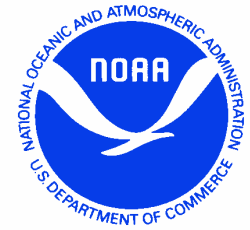
Parameter that governs the median rate of decline.

“Process error”: parameter that describes the long-term variability of the process.



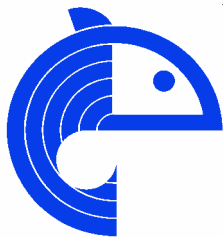
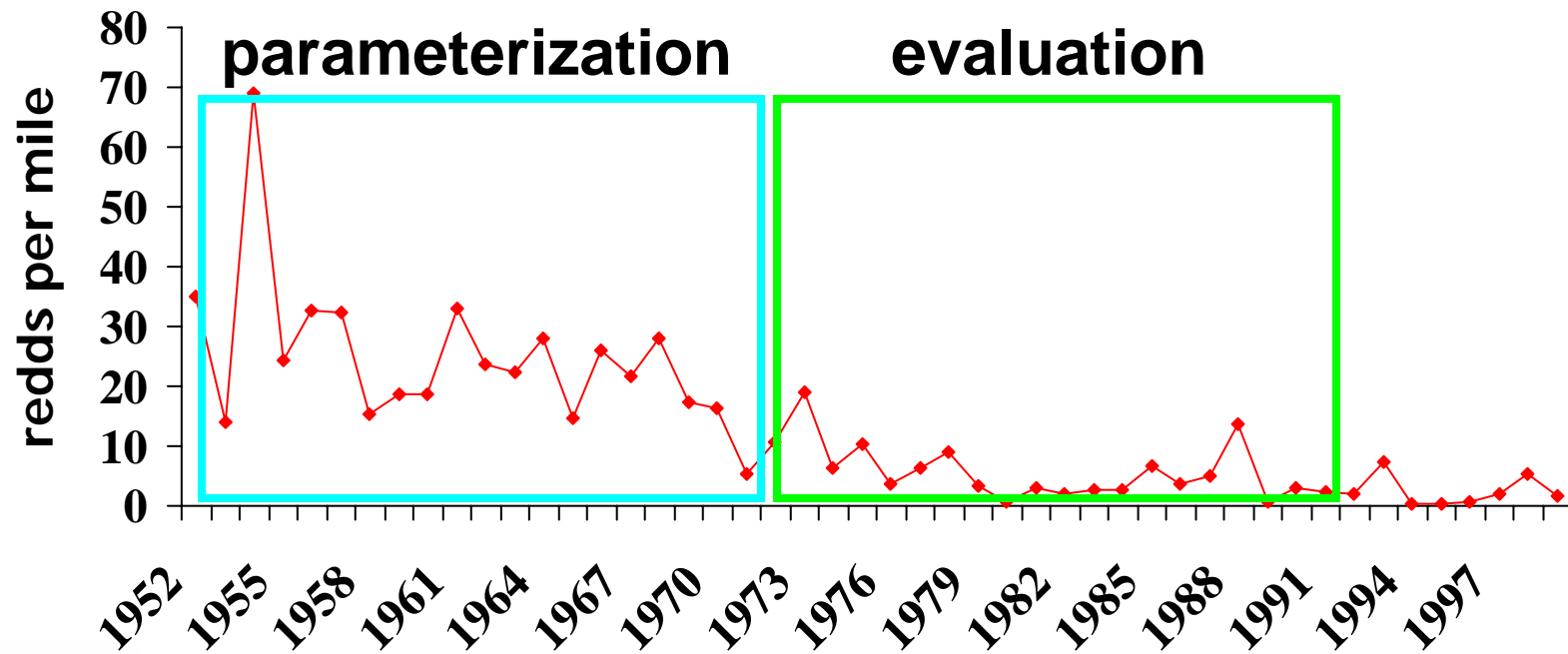
$\hat{\mu}$

$\hat{\sigma}$



# Cross-validation

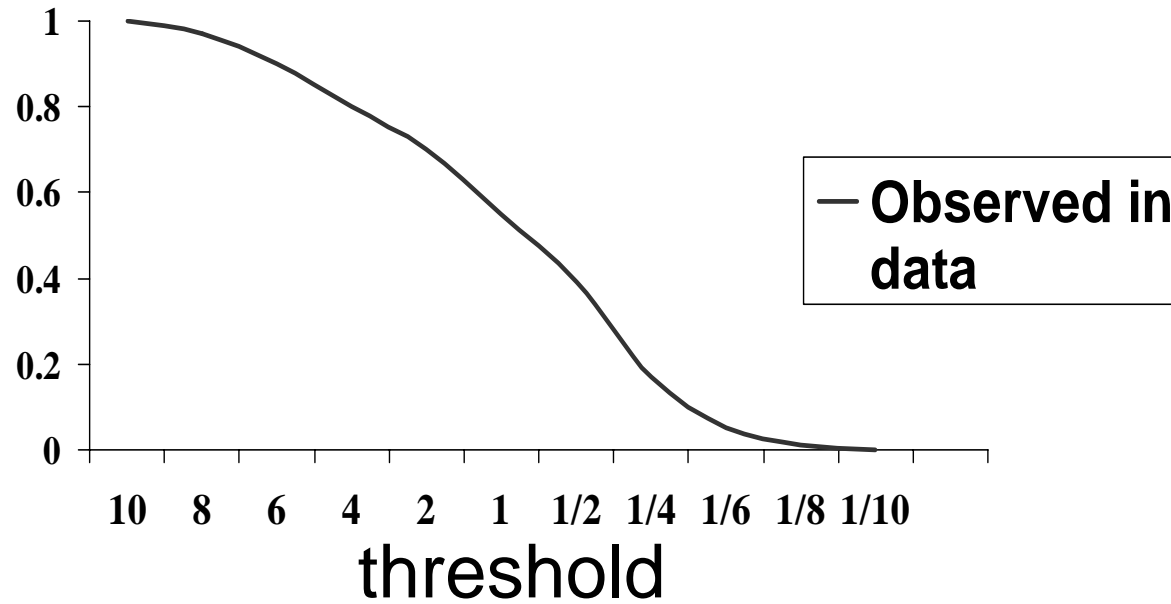
- 147 chinook and 42 steelhead 30-70 year time series from ESUs in WA, OR, and CA





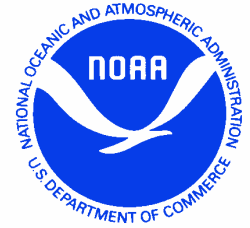
# Does the DA model predict the frequency of actual declines?

fraction of stocks that dip below a threshold

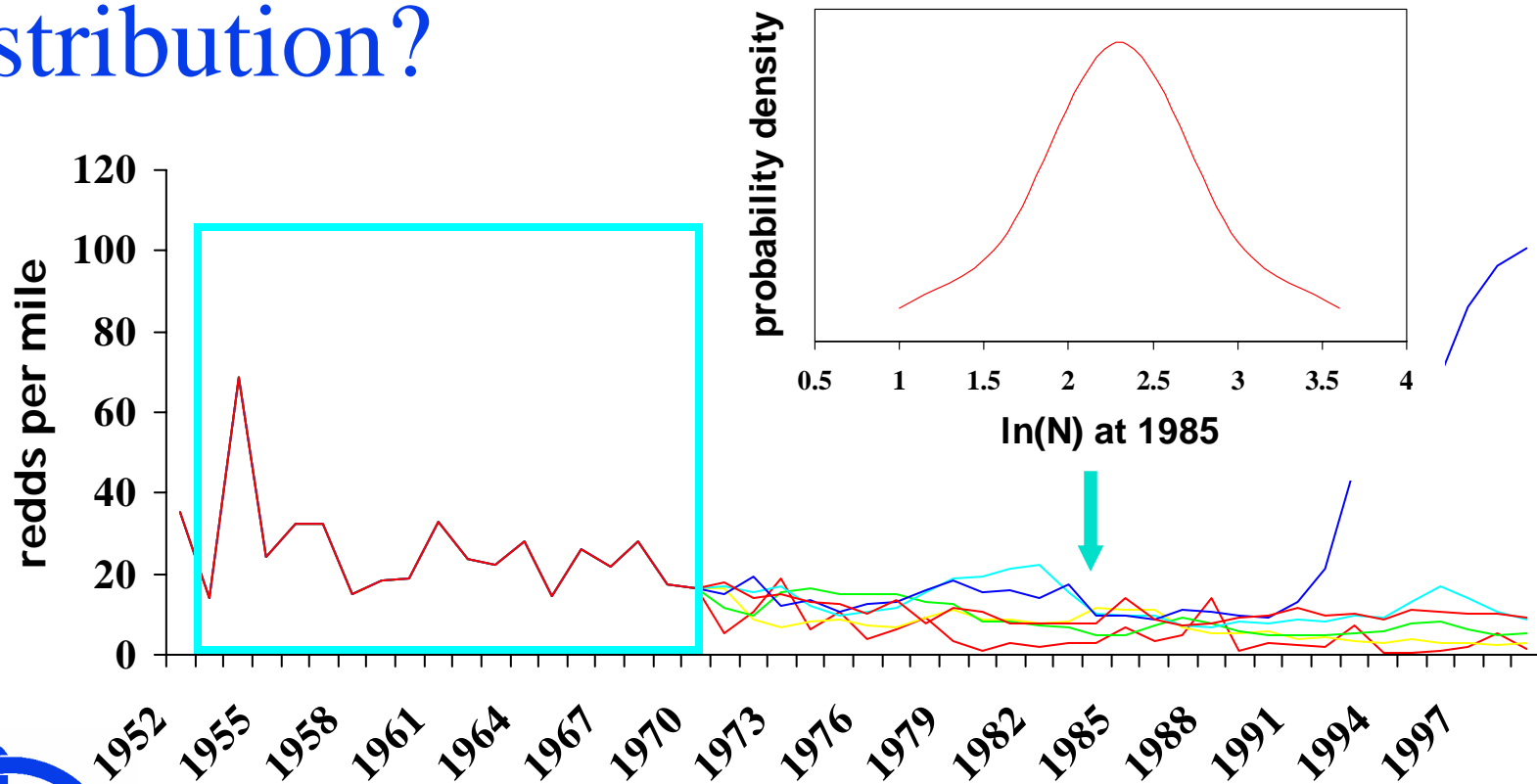


(rel. to size at the start of the eval. period)

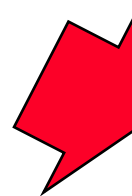
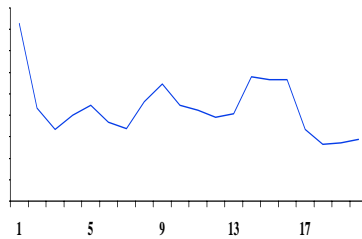
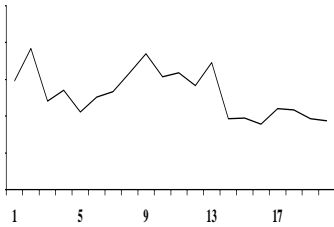
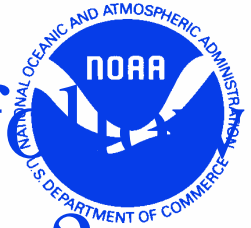




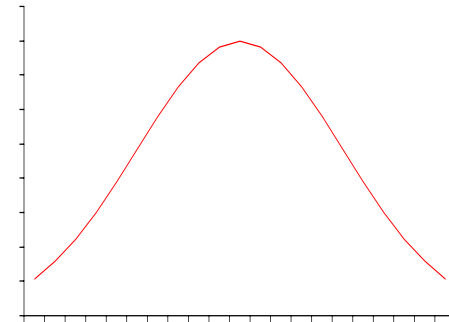
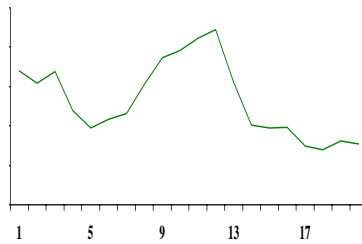
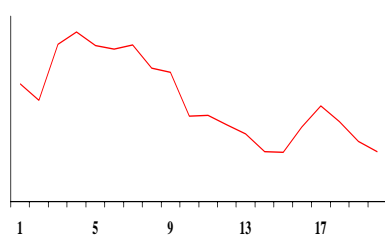
# Do the projected population sizes follow the expected theoretical distribution?

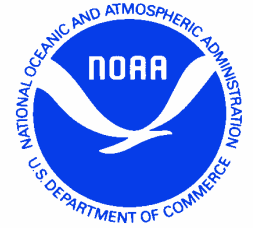


# Do the projected population sizes for the expected theoretical distribution?

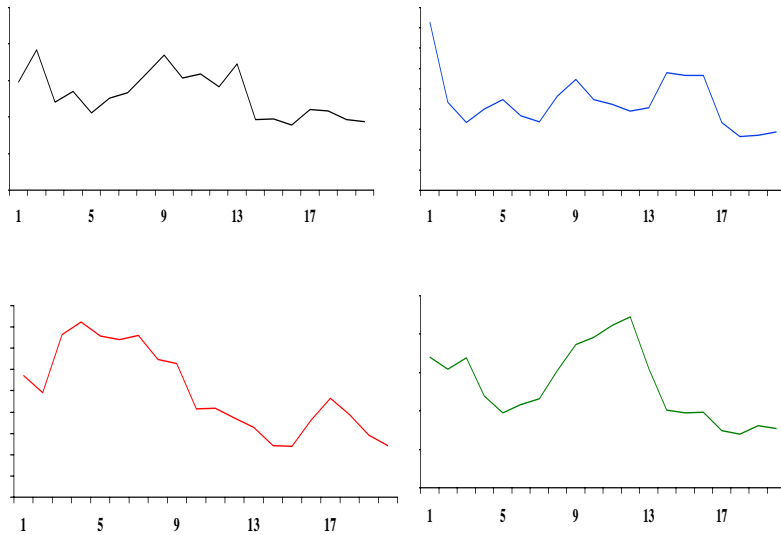


dist. of  $\hat{\mu}$

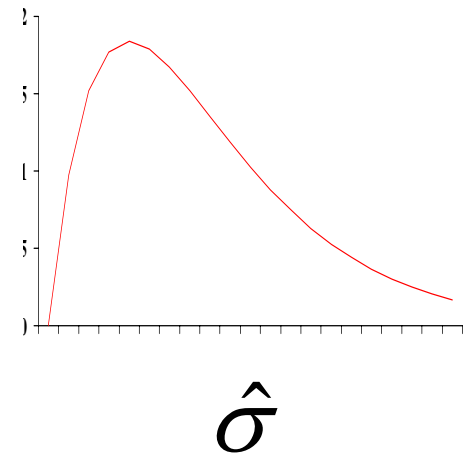


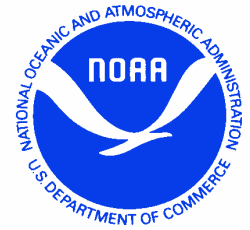


# Do my estimates of $\sigma$ follow the expected theoretical distribution?



dist. of  $\hat{\sigma}$

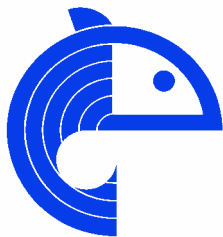




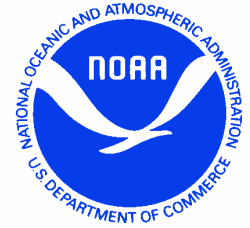
# Transforming data to a common currency

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- **Problem:** don't view the same population process over and over
- **Actual data:** many different processes with different underlying parameters (growth rates and variability)
- **Solution:** transform data to a standardized metric that has the same statistical distribution for all processes



For other examples see McCarthy et al. 2002.  
Conservation Biology

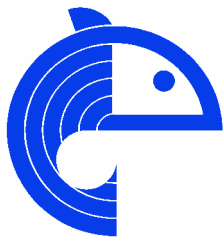


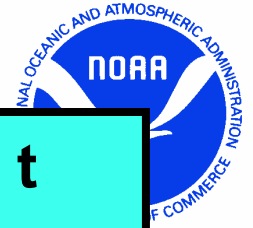
# Standardized $\ln(N_{t+15}/N_t) = \psi$ distribution

$$\frac{(\psi_p - \psi_e)}{\sqrt{\frac{df_{slp} \hat{\sigma}_p^2 + df_{slp} \hat{\sigma}_e^2}{2df_{slp}} \left( \frac{1}{n_p - L} + \frac{1}{n_e - L} \right)}} \sim \frac{1}{\sqrt{\lambda}} t_{2df_{slp}} - 0$$

no bias  $\hat{\mu} \sim$  Normal-like  
of  $\text{var}(\hat{\mu})$  is  $df_{slp}^2$  like

no trend

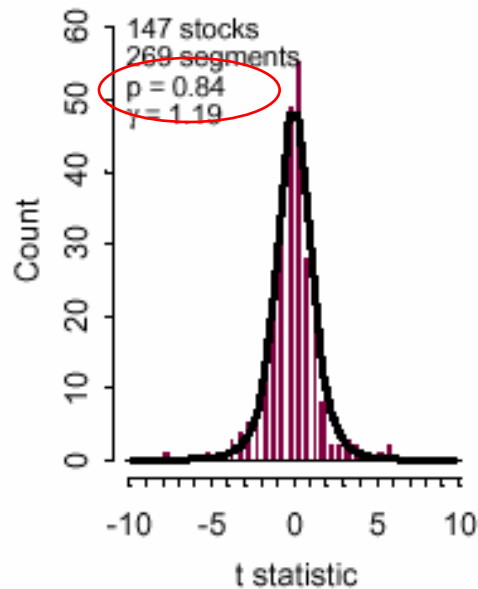




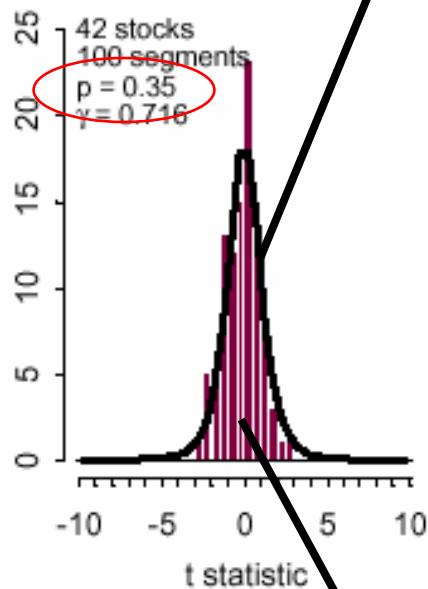
# Results for population distribution

**Predicted t distribution**

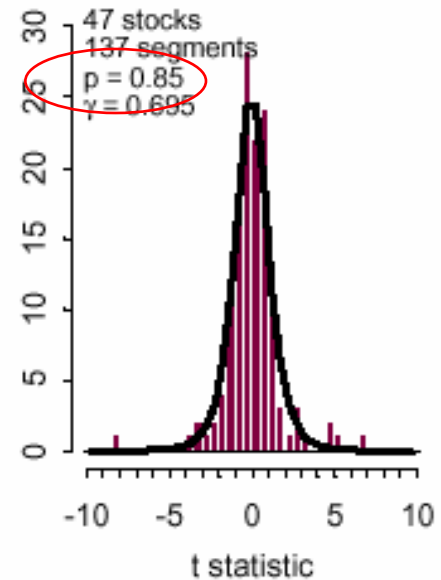
Chinook



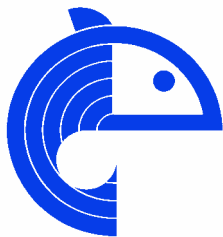
Steelhead

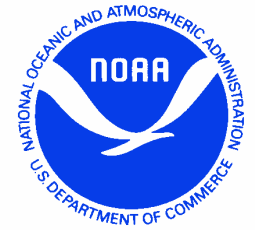


Snake R Spr/Sum



**Histogram of actual t statistics**





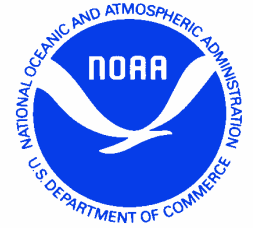
# Trend in the rate of decline?

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- ∪ Fluctuating or declining stocks
  - ∪ No significant trend
- ∪ Rapidly increasing stocks
  - ∪ Significant negative trend
  - ∪ Estimate of  $\mu$  lower for bigger population size



# Standardized $\sigma$ distribution



$$\left(\hat{\sigma}_e^2 / \hat{\sigma}_p^2\right) \sim F(df_{slp}, df_{slp}) \times 1$$

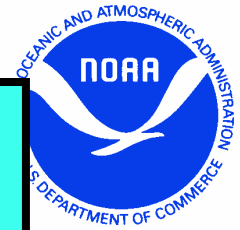
$\hat{\sigma}^2 \sim \chi^2$  - like  
degrees of freedom

no trend

of  $\hat{\sigma}_p^2$  is  $df_{slp}$

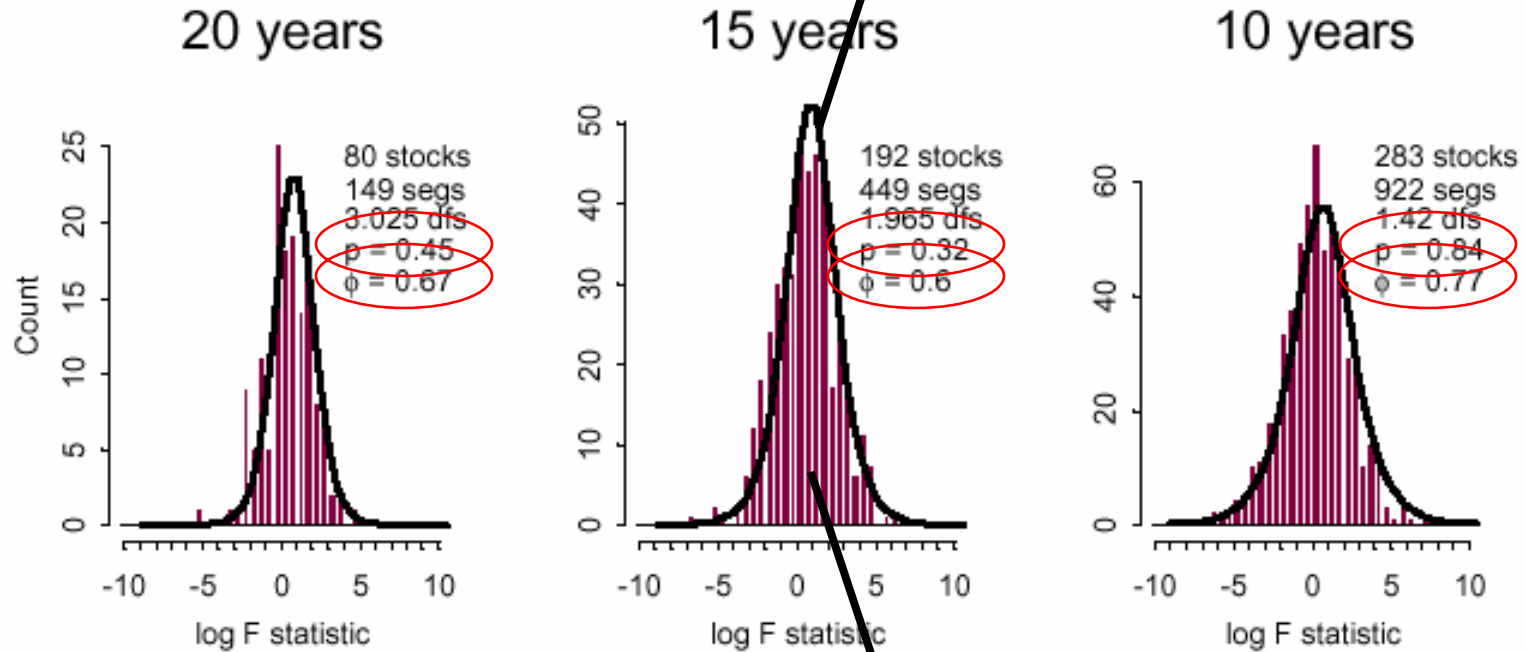




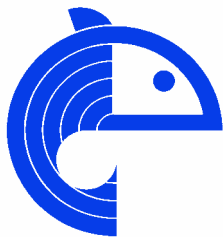


# Results for $\sigma$

**Predicted F distribution**



**Histogram of actual F statistics**





# Trend in $\sigma$ ?

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- u Estimate of  $\sigma$  was higher when counts were really small

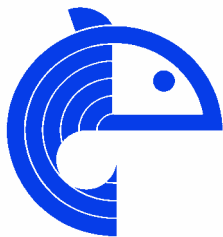
- u Demographic stochasticity?

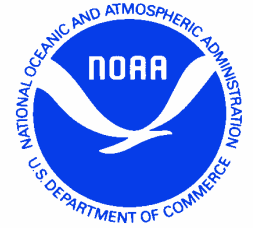
- u Sampling effect?

- Estimate of  $\sigma$  sensitive to percent of sampling error in the observation

- Percent error tends to be larger when counts are small

- e.g. Dunham and Rieman. 2001. Sources and magnitudes of sampling error in redd counts for Bull Trout. *North American Journal of Fisheries Management* 21:343–352



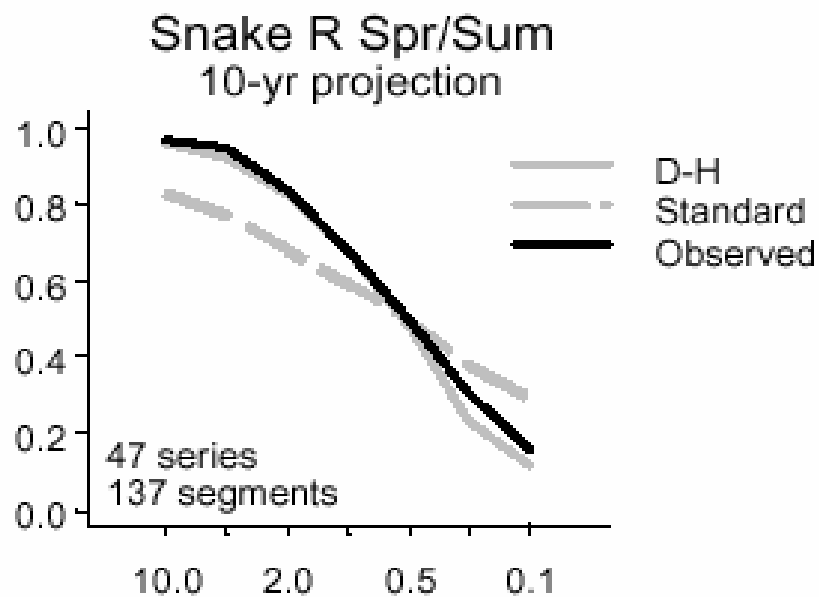
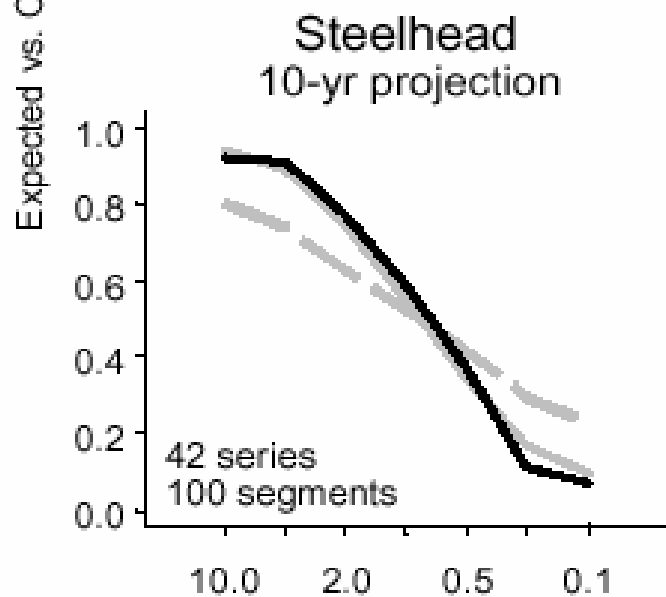
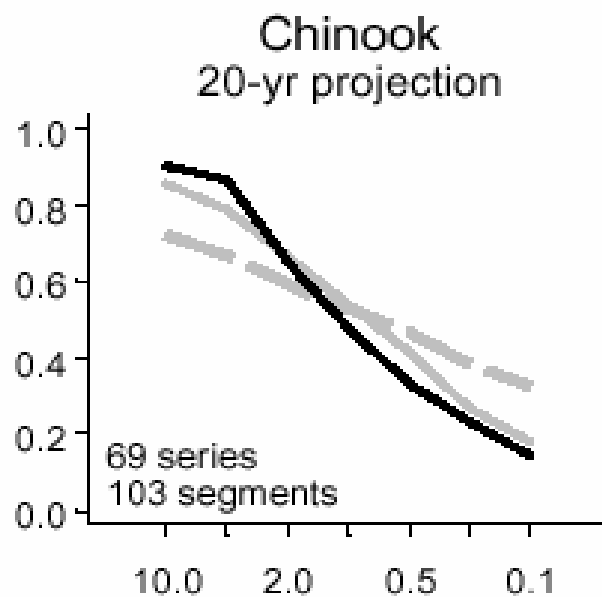
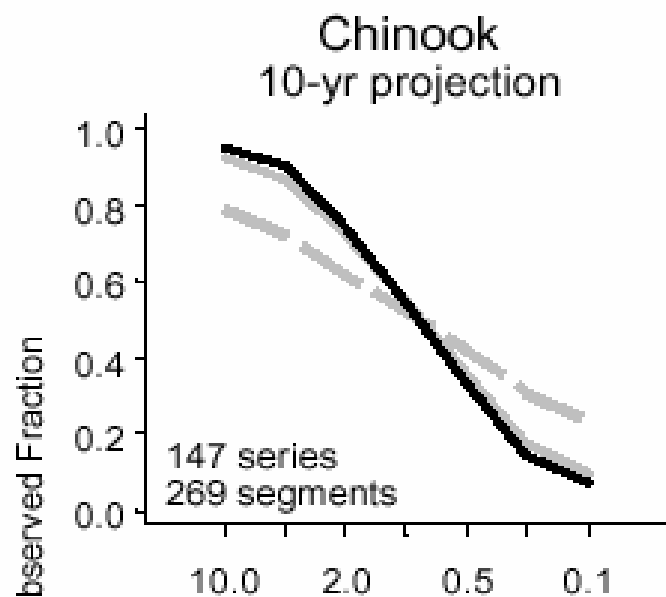


# Can the model predict actual declines?

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- u Simple model makes many simplifying assumptions
  - u density-independence
  - u no environmental correlation
  - u no trends
  - u diffusion approximation of age-structured population



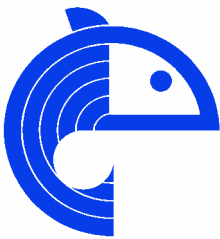
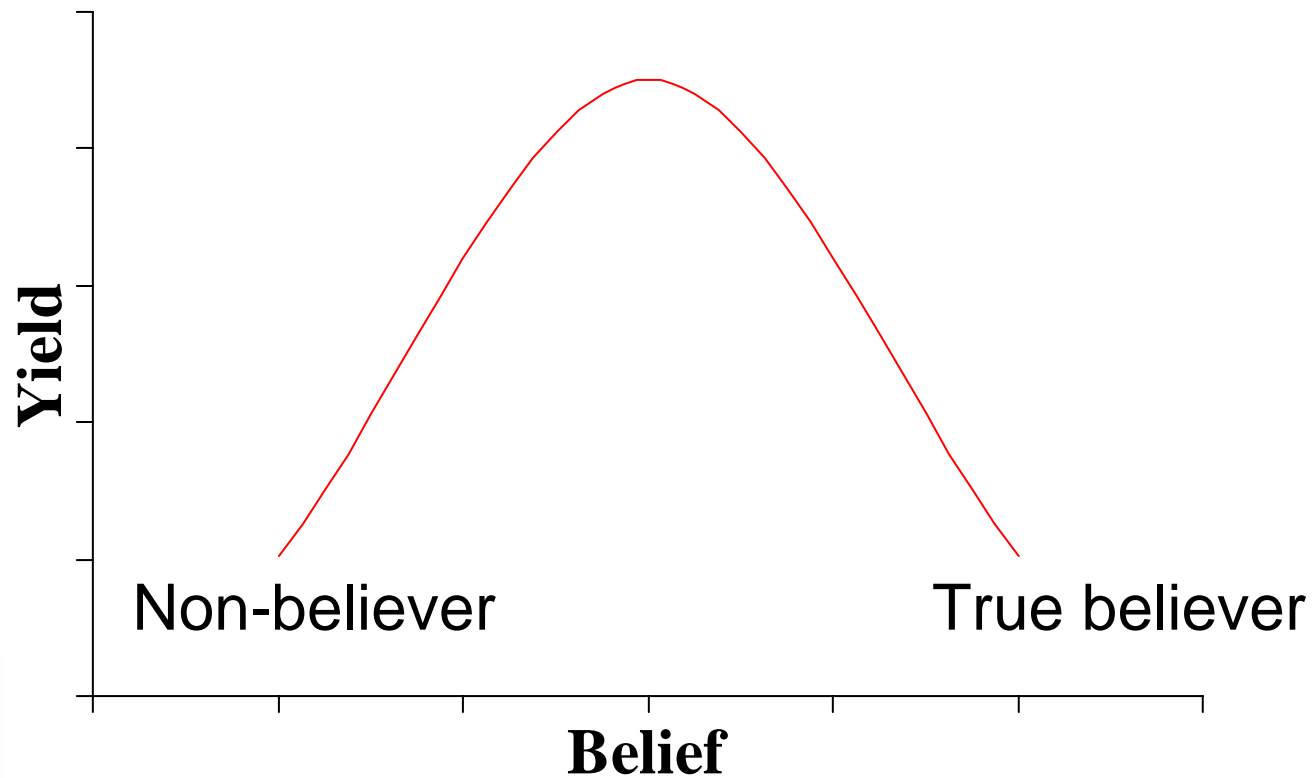
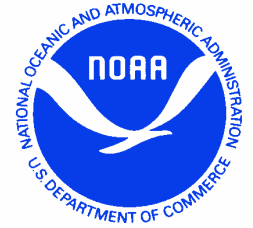


— D-H  
- - Standard  
— Observed

Population Size Threshold (x)

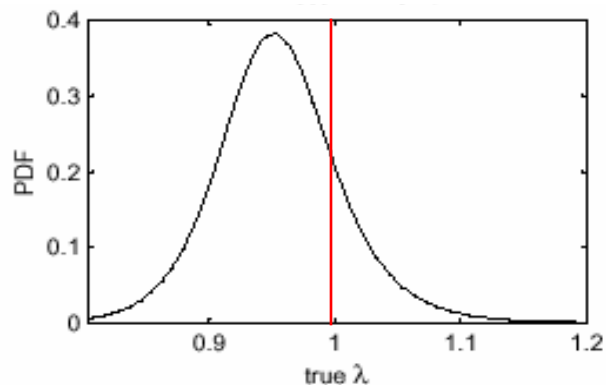
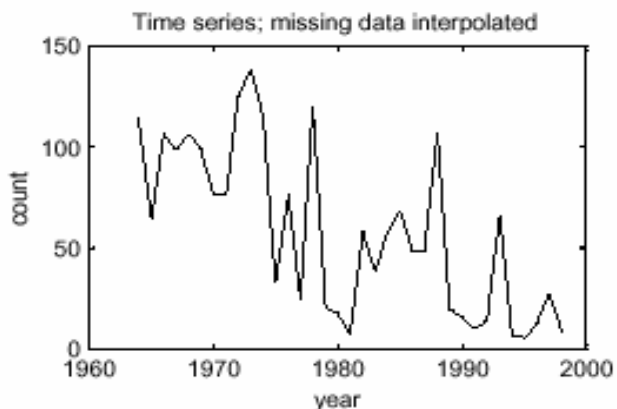
# Maximum yield relationship for modeling

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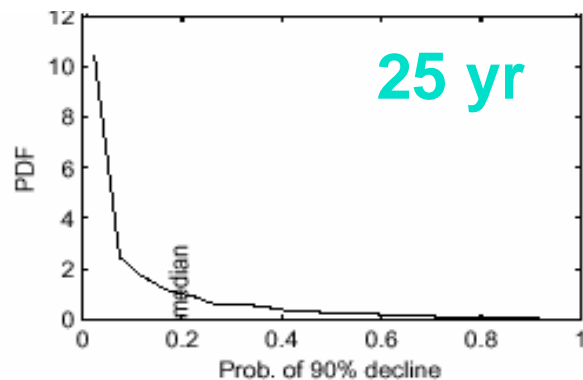
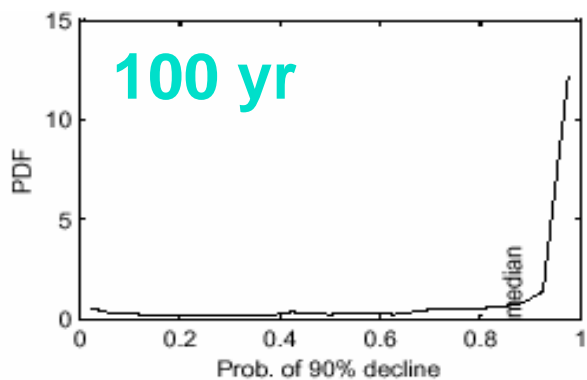


# Lostine River 1964-1998

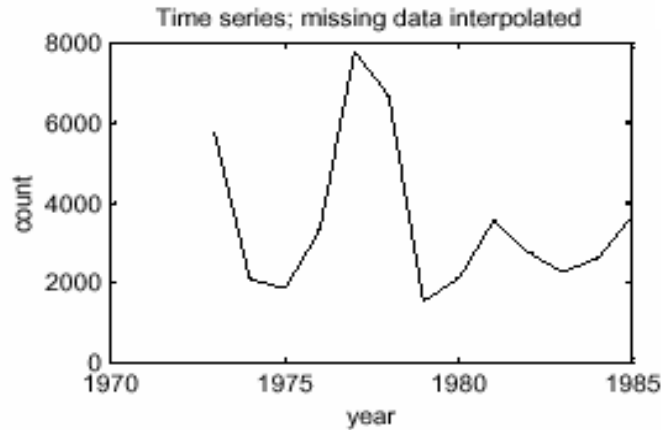
## support for different $\lambda$ 's



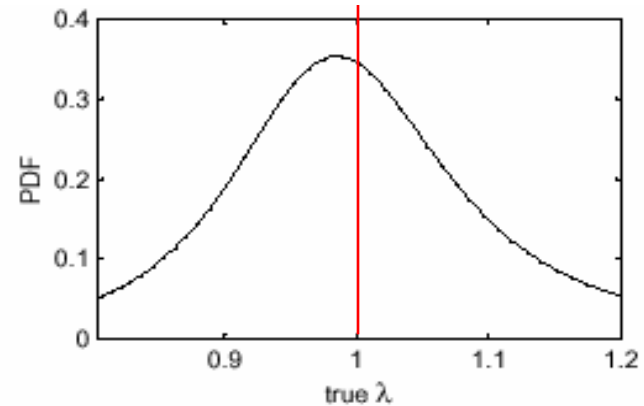
## support for different probs of 90% decline



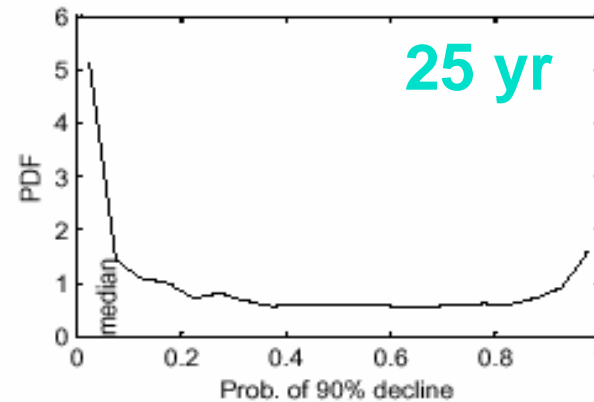
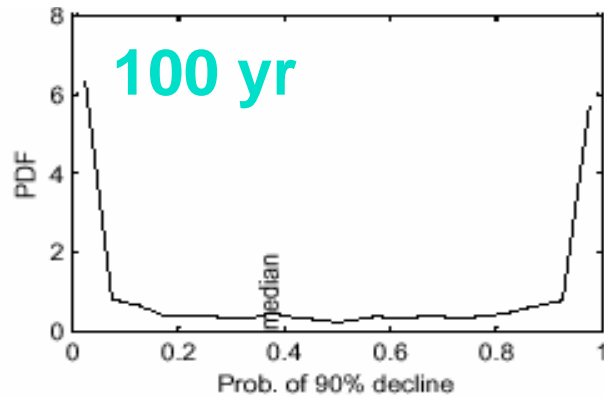
## Clearwater River 1973-1985



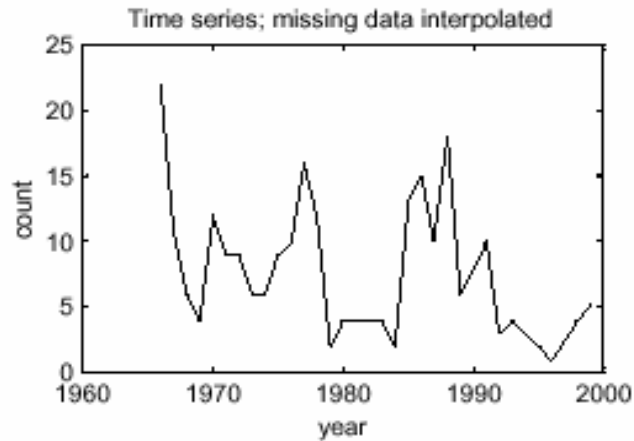
## support for different $\lambda$ 's



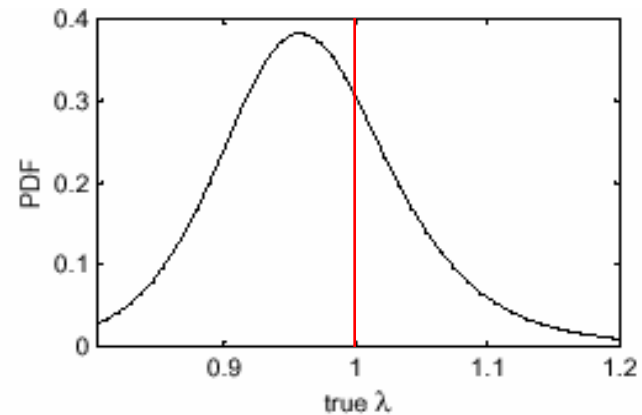
## support for different probs of 90% decline



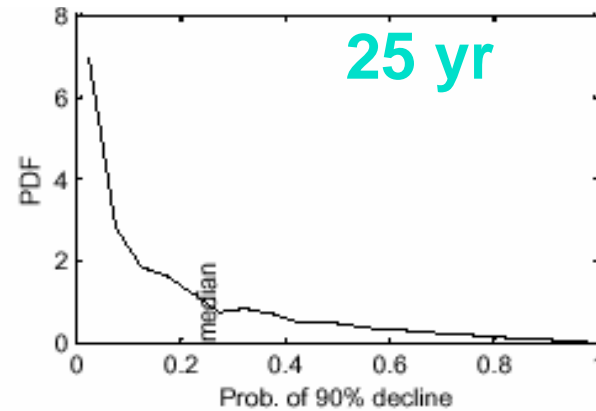
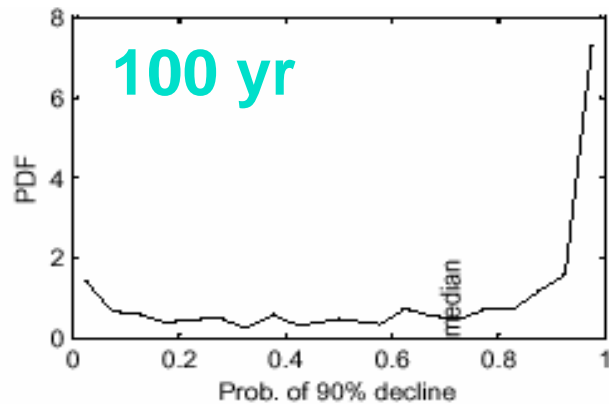
## Camp Creek 1966-1999



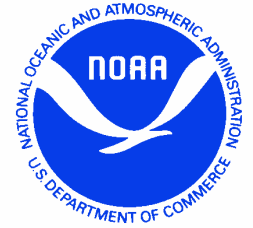
## support for different $\lambda$ 's



## support for different probs of 90% decline







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## FOR MORE INFO...

**A variety of matlab and Splus code for DA PVAs is at  
[faculty.washington.edu/eeholmes](http://faculty.washington.edu/eeholmes)**

**Holmes, E. E. 2001. Estimating risks in declining  
populations with poor data. Proceedings of the National  
Academy of Science 98: 5072-5077.**

**Holmes and Fagan. 2002. Validating population viability  
analysis for corrupted data sets. Ecology in press.**

**Holmes, E. E. Beyond theory to application and evaluation:  
diffusion approximations for population viability  
analysis.**



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