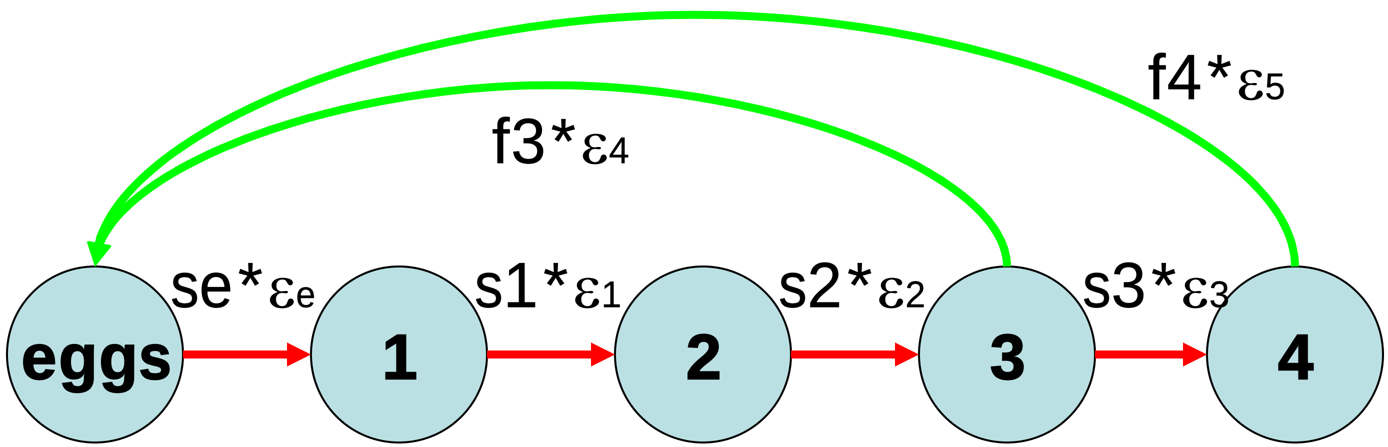
diffusion approximation approaches for metapopulation viability analysis

‘stochastic approximation’ approach: example from single population



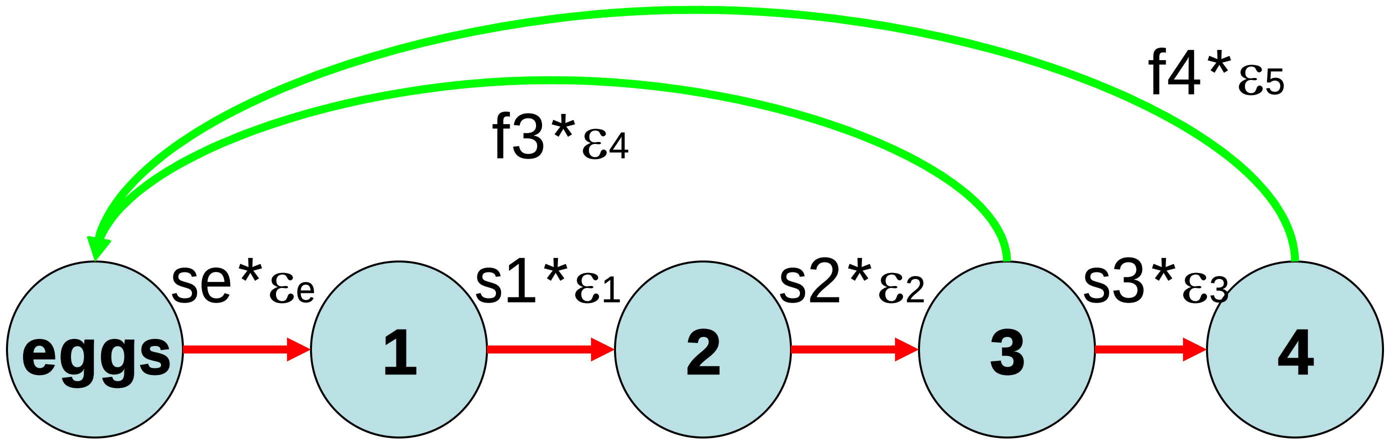
= **Asymptotic stochastic model**

$$\ln N_{t+\tau} = \ln N_t + \mu\tau + \varepsilon, \quad \varepsilon \sim N(0, \sigma\, \mathrm{sqrt}(\tau))$$

≈ **Markov or diffusion approximation**

$$\ln N_{t+1} = \ln N_t + \mu + \varepsilon, \quad \varepsilon \sim N(0, \sigma)$$

corrupted diffusion model



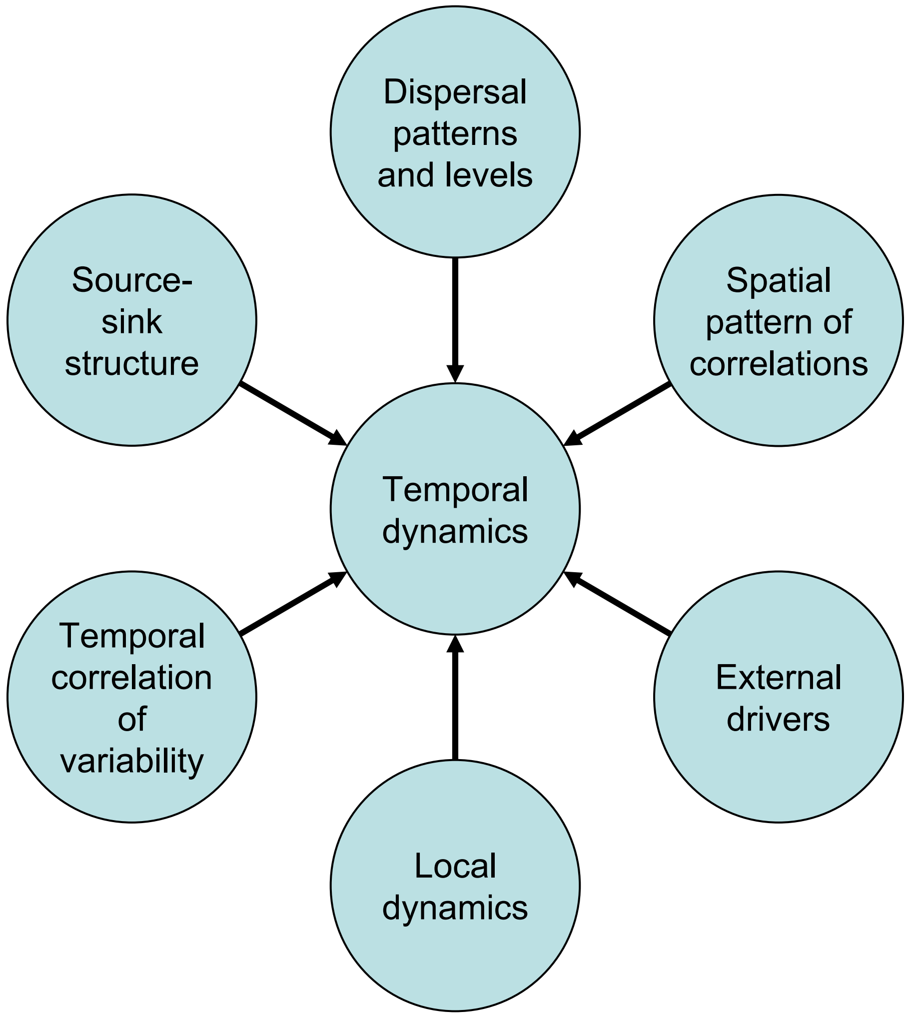
+ **Various types of observation and other extraneous variance**

≈ **state-space model**

$$\ln N_{t+1} = \ln N_t + \mu + \varepsilon, \quad \varepsilon \sim N(0,\sigma)$$

$$\ln Y_{t+1} = \ln Y_{t+1} + \varepsilon_s, \ \varepsilon_s \sim f(\nu,\sigma_s)$$

lots of things affect metapopulation dynamics



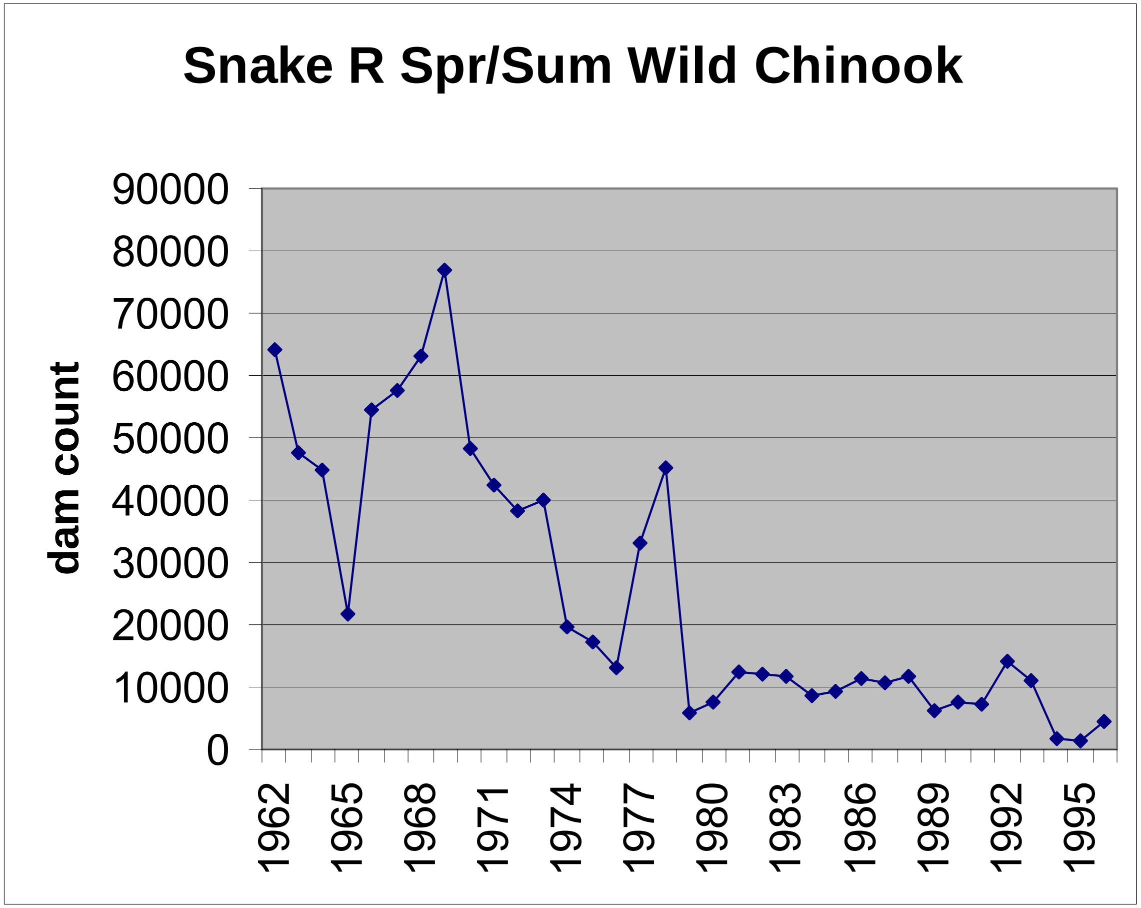
IUCN Red List Criteria

- Criteria A2: “A reduction of at least xx%, projected or suspected to be met within the next xx years….”
- Criteria C1: “Population estimated to number less than xx and an estimated continuing decline of at least xx% within xx years....”
- Criteria E: “Quantitative analysis showing the probability of extinction in the wild is at least xx% within xx years...”

diffusion approximations for metapopulations

- A theoretical framework for ‘diffusion approximations’ for metapopulations
- Application to declining metapopulations
- Example using chinook salmon
- Some conjectures regarding management implications

focus on declining metapopulations



local internal dynamics

$$N_i(t) = N_i(t-1)\underline{e^{z_i(t-1)}}$$ stochastic population growth

$$-\underline{d_i(t-1)}N_i(t-1)\underline{e^{z_i(t-1)}}$$ dispersal out

$$+\sum_{j\neq i}\underline{\alpha_{ji}(t-1)}\,\underline{d_j(t-1)}\;N_j(t-1)\underline{e^{z_j(t-1)}}$$ dispersal in

- Any spatial pattern of dispersal (any kernel)
- Any temporally varying distributions of dispersal rates as long as the distributions are stationary
- Any spatial pattern of correlation in growth rates among sites
- z's need not be normally distributed

Matrix representation of stochastic metapopulation

$$\begin{bmatrix} N_1(t+1) \\ N_2(t+1) \\ N_3(t+1) \\ \dots \\ N_k(t+1) \end{bmatrix} = \mathbf{A}(t) \times \begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \\ \dots \\ N_k(t) \end{bmatrix}$$

stochastic matrix

$$\mathbf{A}(t) = \begin{bmatrix} (1-d_1)e^{z_1} & \alpha_{21}d_2e^{z_2} & \alpha_{31}d_3e^{z_3} & \dots & \alpha_{k1}d_ke^{z_k} \\ \alpha_{12}d_1e^{z_1} & (1-d_2)e^{z_2} & \alpha_{32}d_3e^{z_3} & \dots & \alpha_{k2}d_ke^{z_k} \\ \alpha_{13}d_1e^{z_1} & \alpha_{23}d_2e^{z_2} & (1-d_3)e^{z_3} & \dots & \alpha_{k3}d_ke^{z_k} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{1k}d_1e^{z_1} & \alpha_{2k}d_2e^{z_2} & \alpha_{3k}d_3e^{z_3} & \dots & (1-d_k)e^{z_k} \end{bmatrix}$$

$$\begin{bmatrix} N_1(t+\tau) \\ N_2(t+\tau) \\ N_3(t+\tau) \\ \dots \\ N_k(t+\tau) \end{bmatrix} = \underline{\mathbf{A}(t+\tau-1)\times\mathbf{A}(t+\tau-2)\times\dots\mathbf{A}(t+1)\times\mathbf{A}(t)}\times\begin{bmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \\ \dots \\ N_k(t) \end{bmatrix}$$

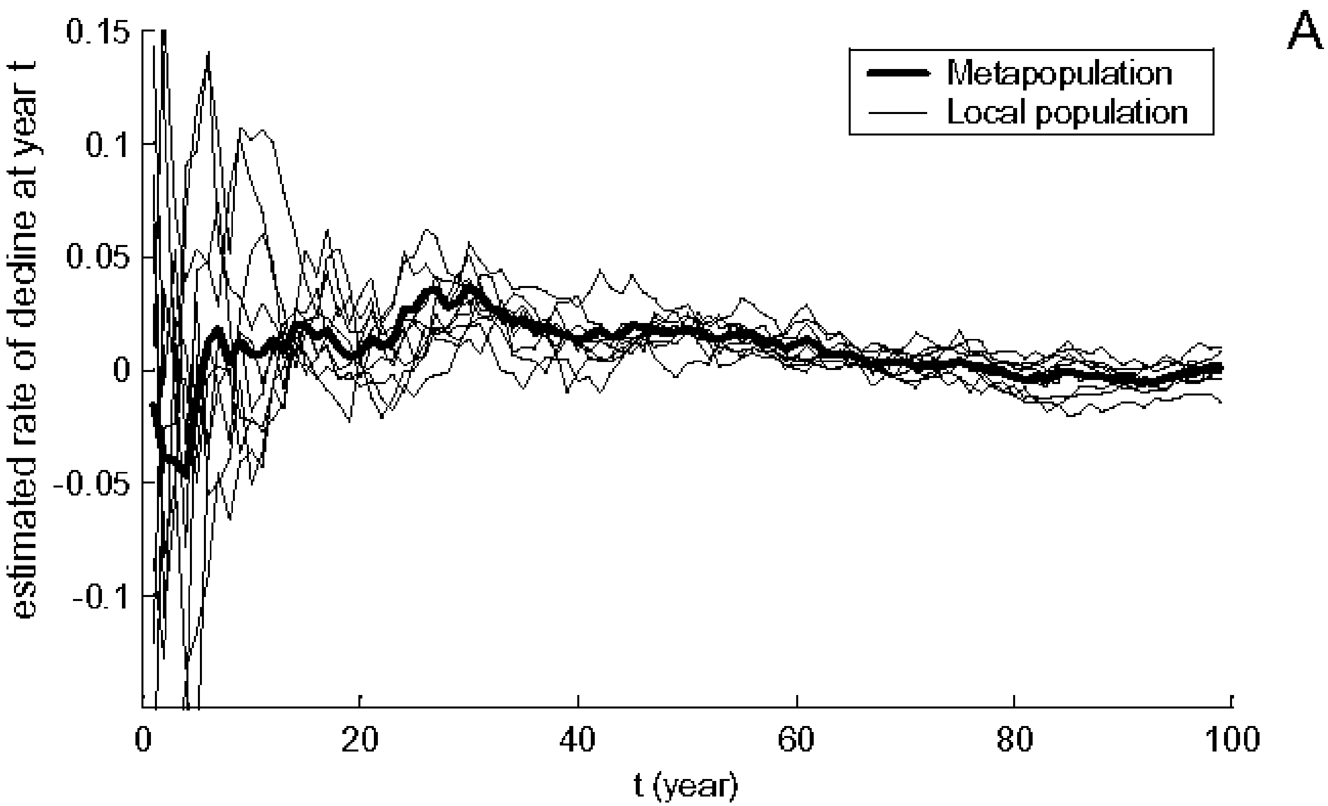
Product of random matrices

$$\ln M(t+\tau) \underset{t\to\infty}{\longrightarrow} \ln M(t) + \tau\mu_m + \varepsilon$$

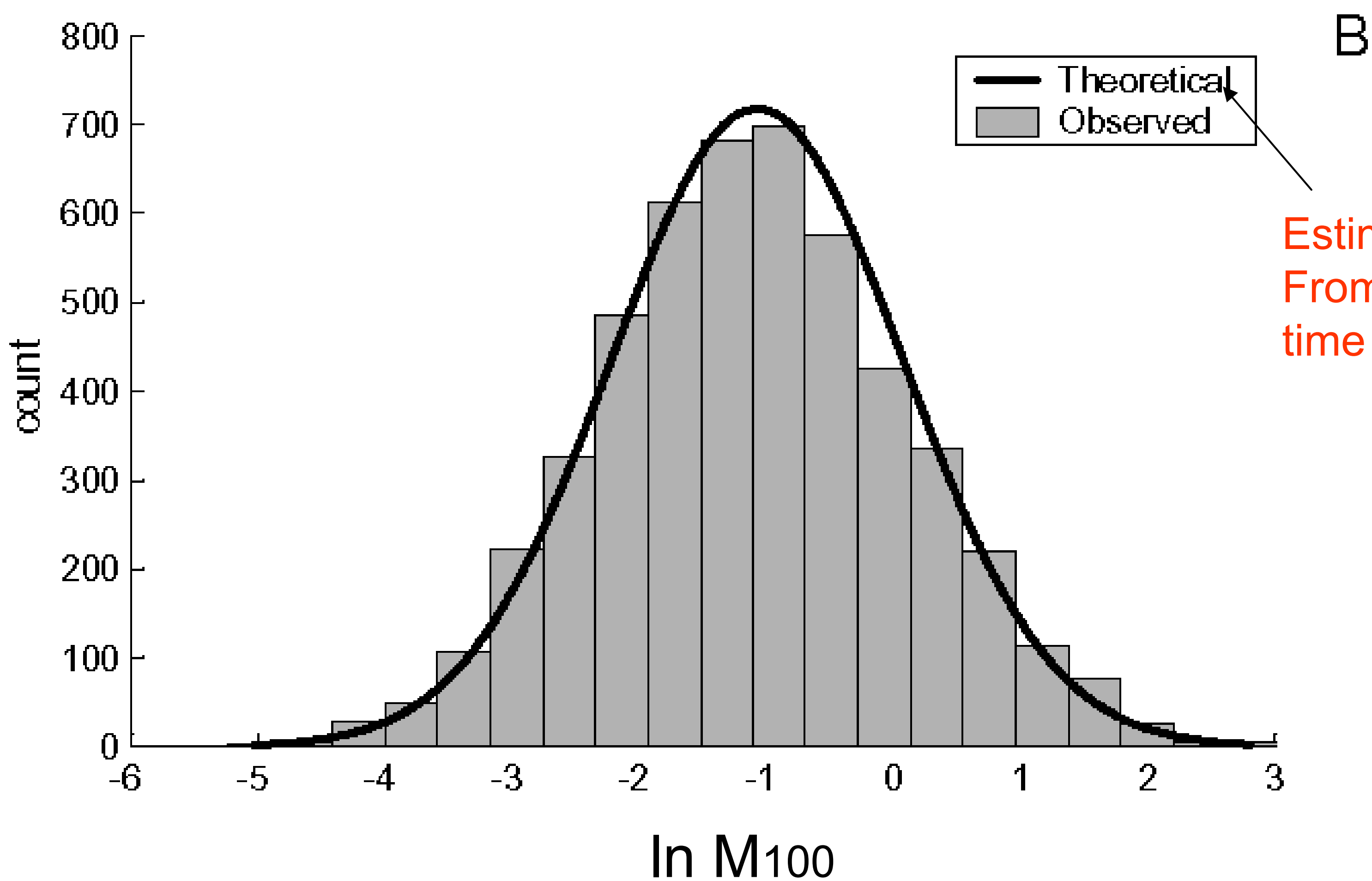
$$\ln N_i(t+\tau) \underset{t\to\infty}{\longrightarrow} \ln N_i(t) + \tau\mu_m + \varepsilon$$

$$\varepsilon \sim normal(0, \sigma\sqrt{\tau})$$

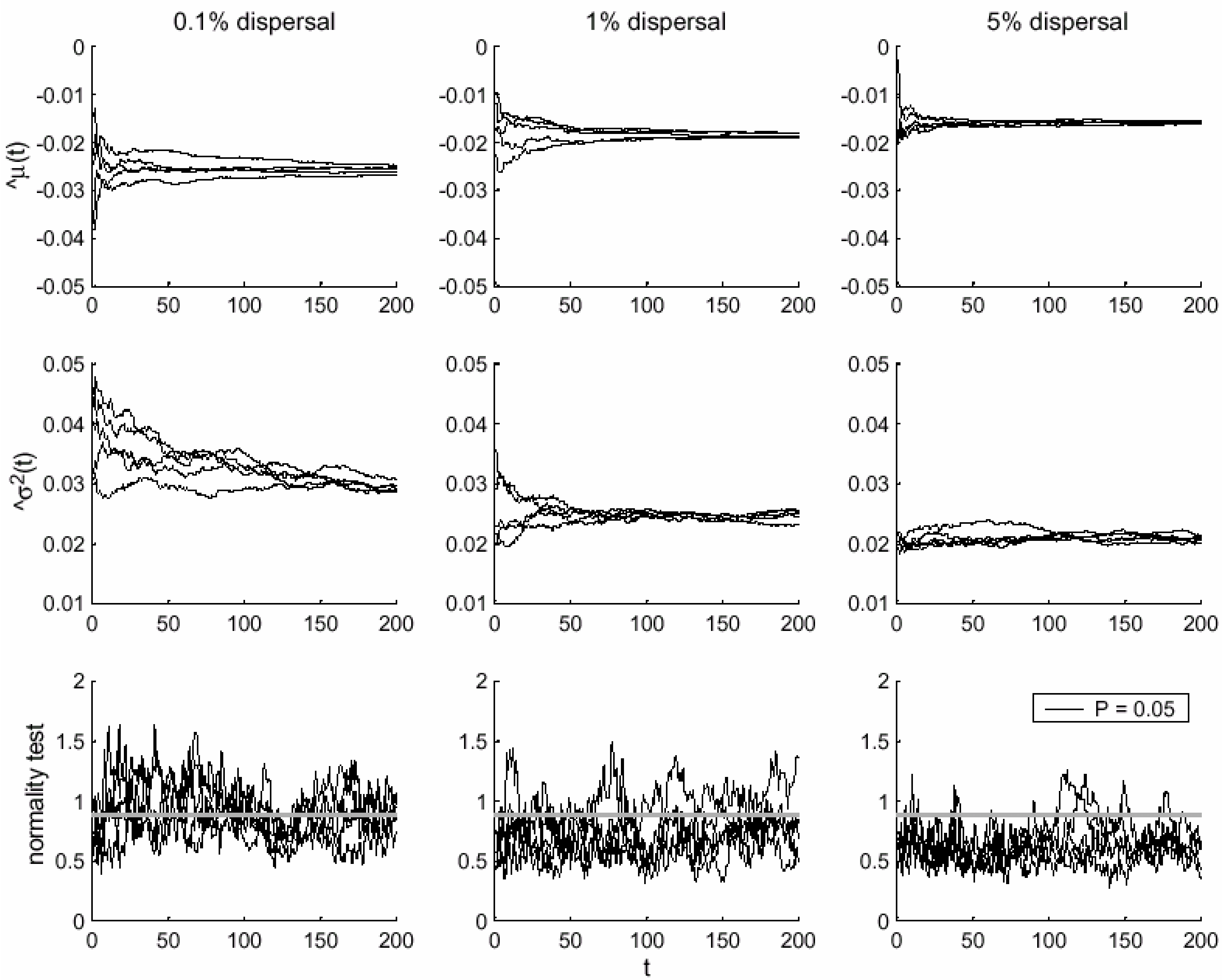
simulated example of long-term dynamics

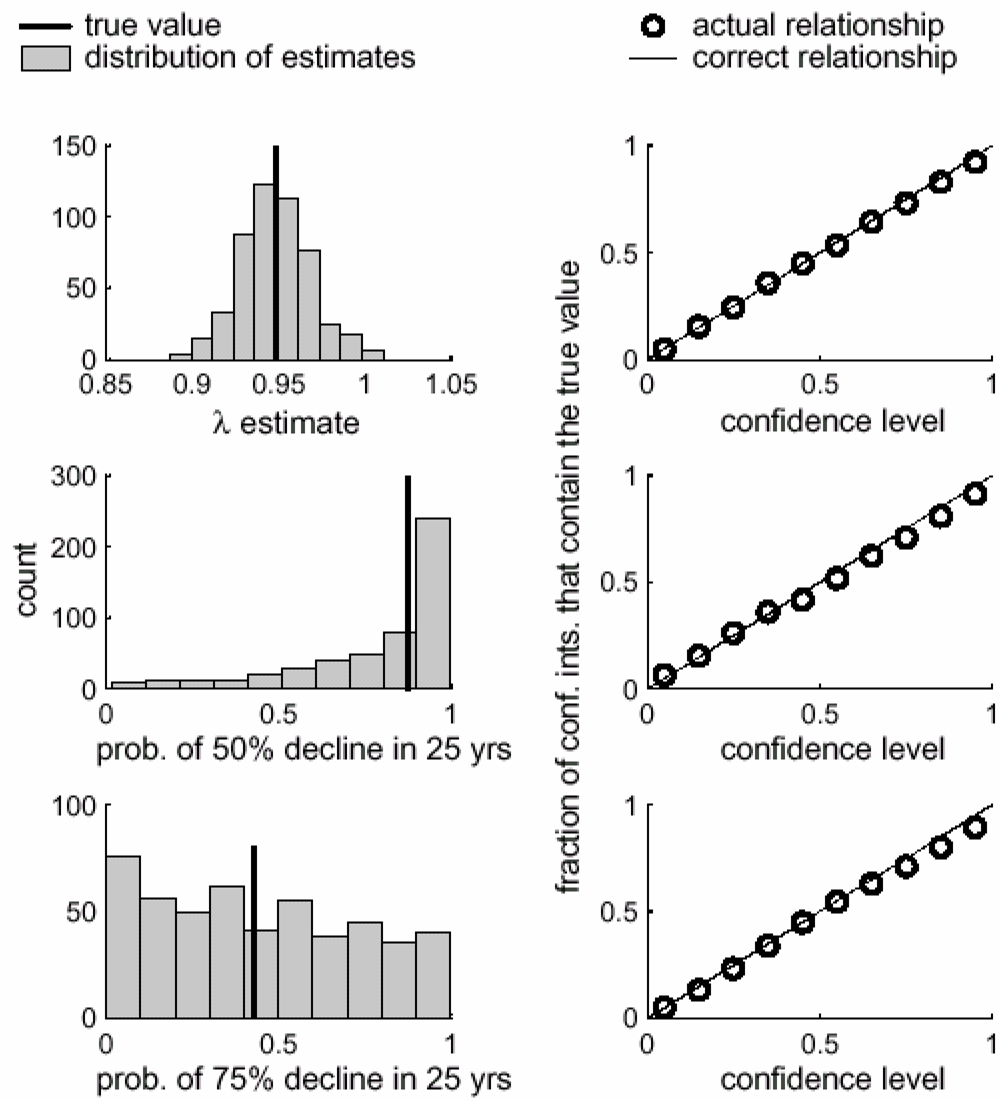


observed vs theoretical distribution of metapopulation size



Estimated
From a single
time series

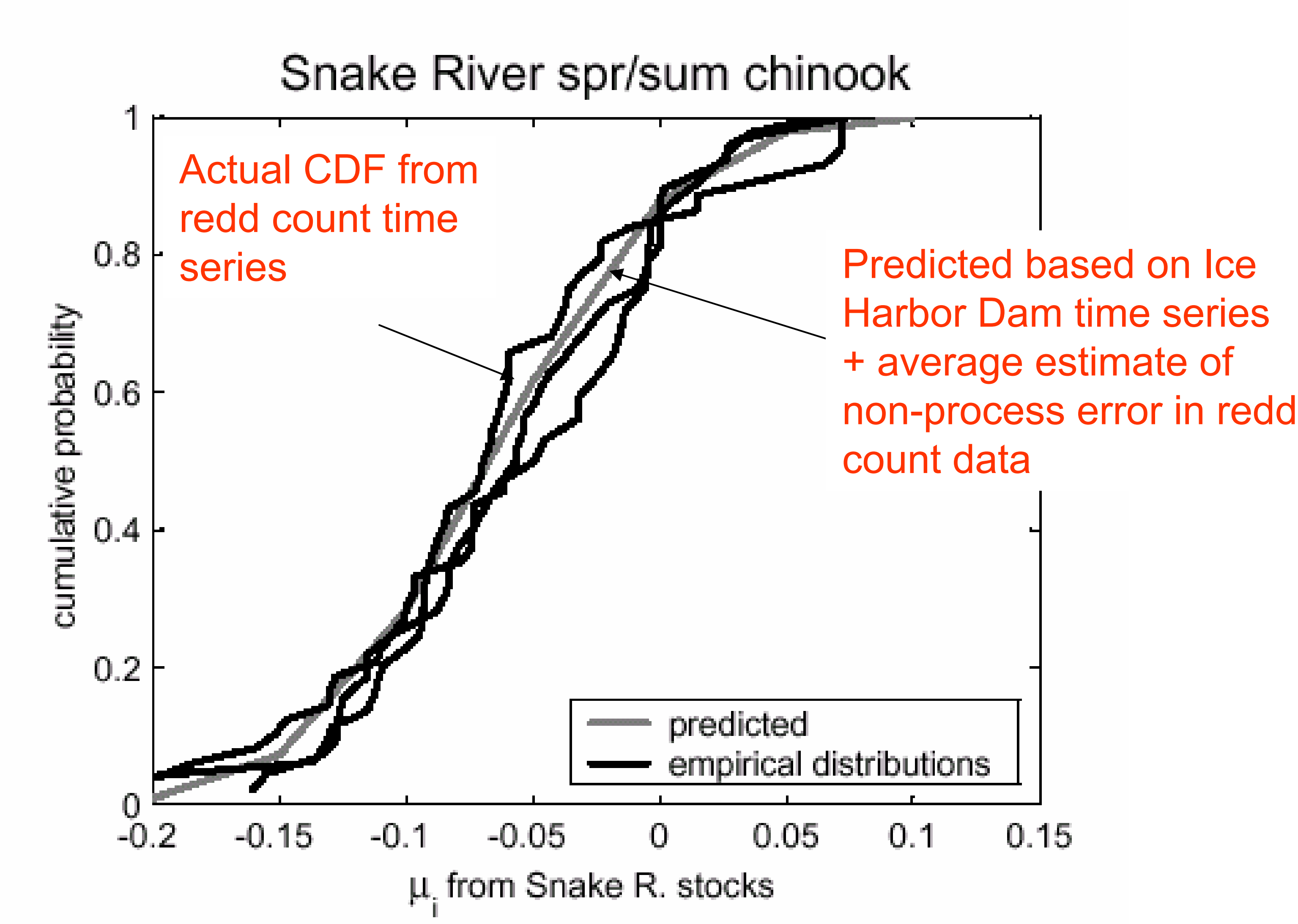

0.1% dispersal
1% dispersal
5% dispersal
^μ(t)
^σ2(t)
normality test
P = 0.05
t


true value
distribution of estimates
actual relationship
correct relationship
count
λ estimate
prob. of 50% decline in 25 yrs
prob. of 75% decline in 25 yrs
fraction of conf. ints. that contain the true value
confidence level

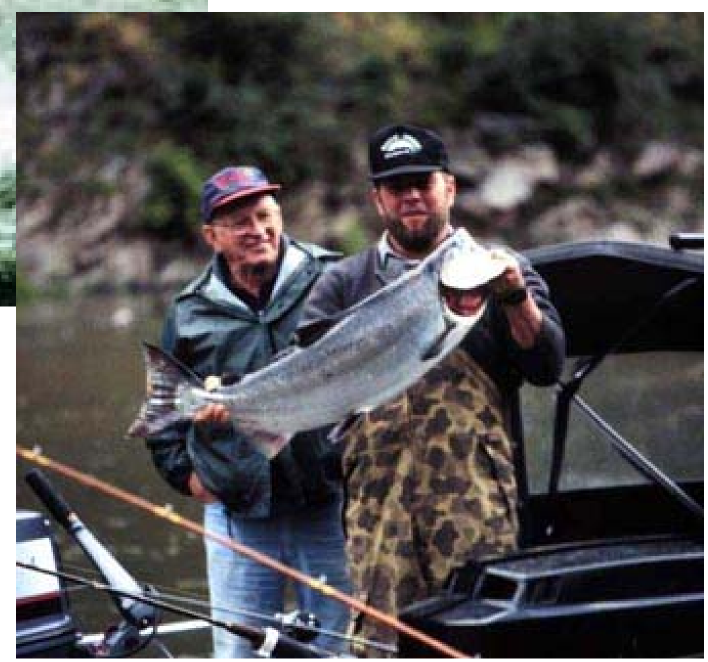
from theory to application

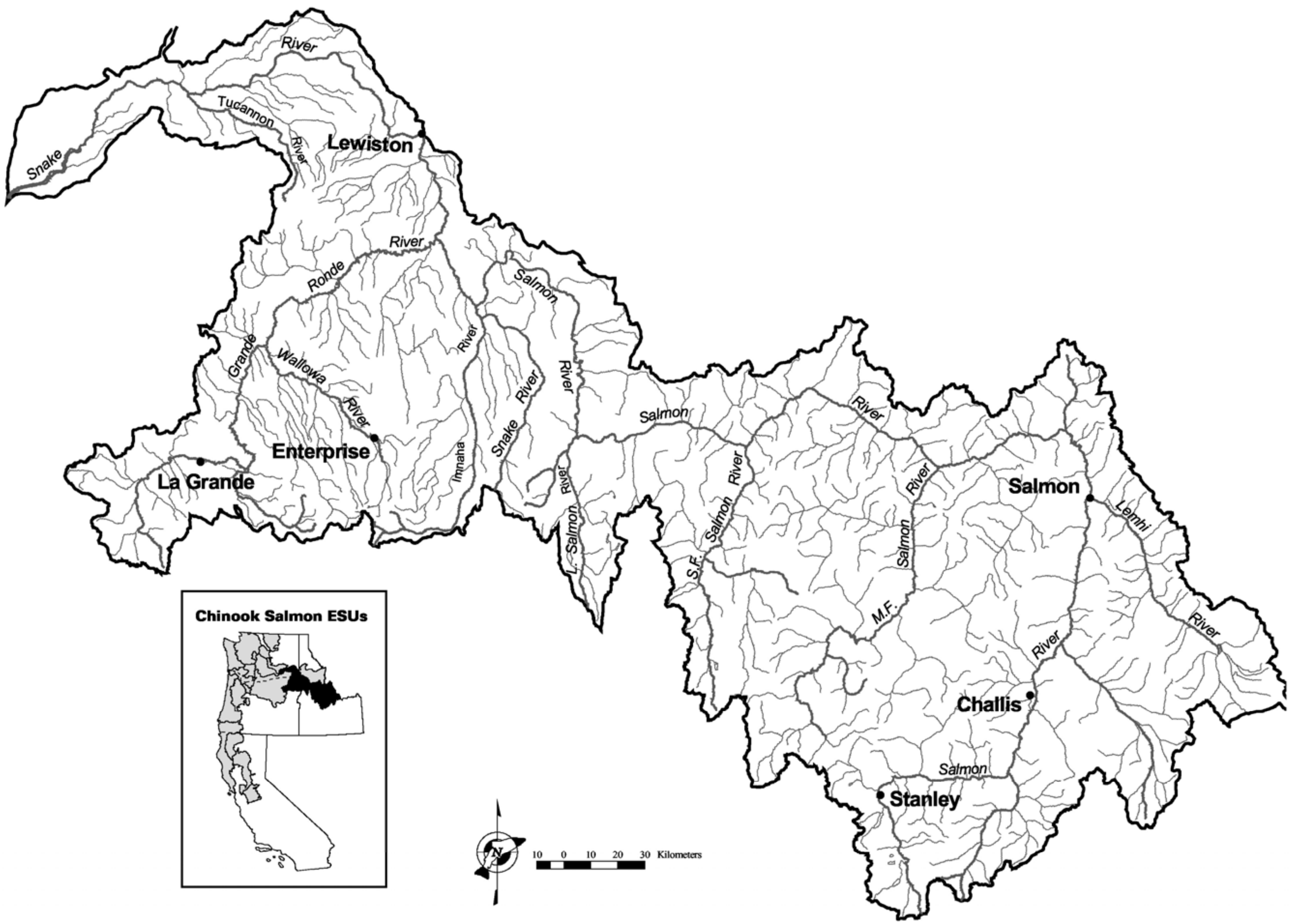
- Develop of approximation: ln M(t)/M(0) = normal(tμ, tσ)
- Develop of statistical methods estimating parameters: μ and σ
- Cross-validation using data
- Use of approximations for PVA purposes

Snake R Spr/Sum Chinook Salmon


Snake River spr/sum chinook
Actual CDF from redd count time series
Predicted based on Ice Harbor Dam time series + average estimate of non-process error in redd count data
cumulative probability
0
0.2
0.4
0.6
0.8
1
-0.2
-0.15
-0.1
-0.05
0
0.05
0.1
0.15
μ_i from Snake R. stocks
predicted
empirical distributions

management conjectures

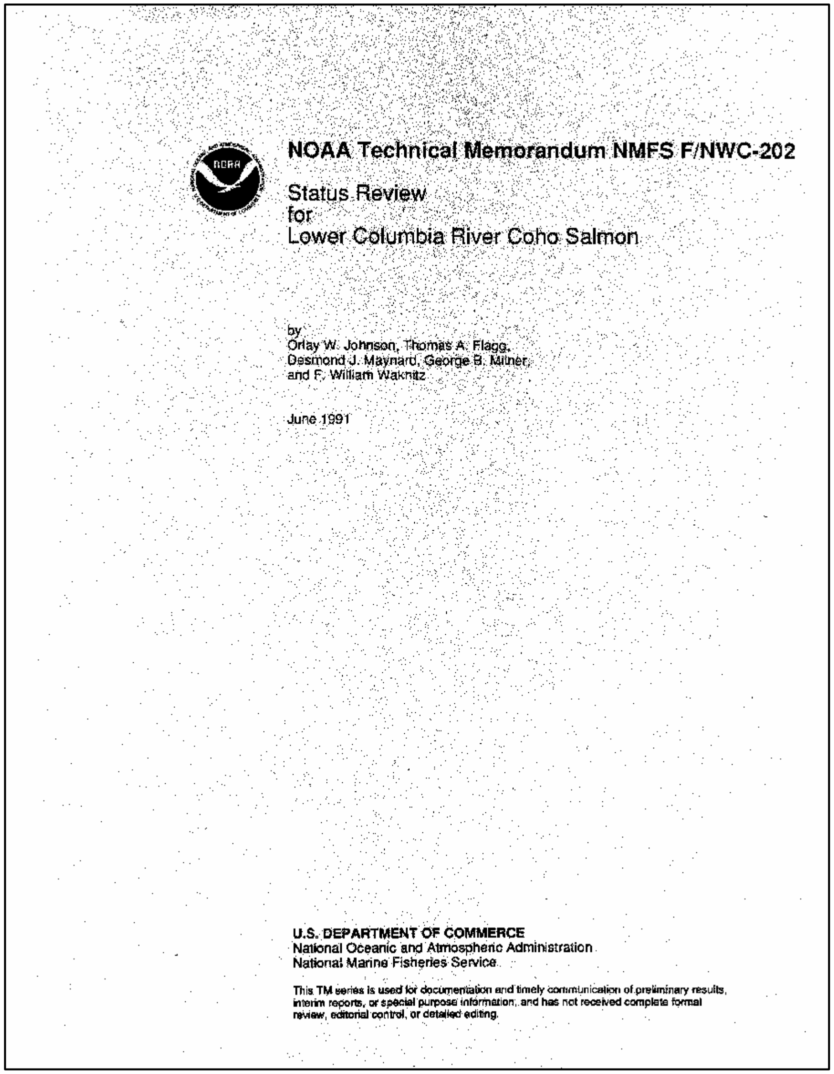




River
Tucannon
River
Lewiston
Snake
River
Ronde
Grande
Wallowa
River
Enterprise
La Grande
Imnaha
River
Snake
River
Salmon
River
River
L. Salmon
Salmon
River
S.F. Salmon
River
Salmon
River
M.F.
Salmon
Lemhi
River
River
Challis
Salmon
Stanley
10 0 10 20 30 Kilometers
Chinook Salmon ESUs

Observed subpop rates of decline do not reflect local conditions

- *At equilibrium*, observed differences in local trends occur by chance
- Actual local conditions (plus connectivity) are reflected in local density


NOAA Technical Memorandum NMFS F/NWC-202

Status Review
for
Lower Columbia River Coho Salmon

by
Orlay W. Johnson, Thomas A. Flagg,
Desmond J. Maynard, George B. Milner,
and F. William Waknitz

June 1991

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Marine Fisheries Service

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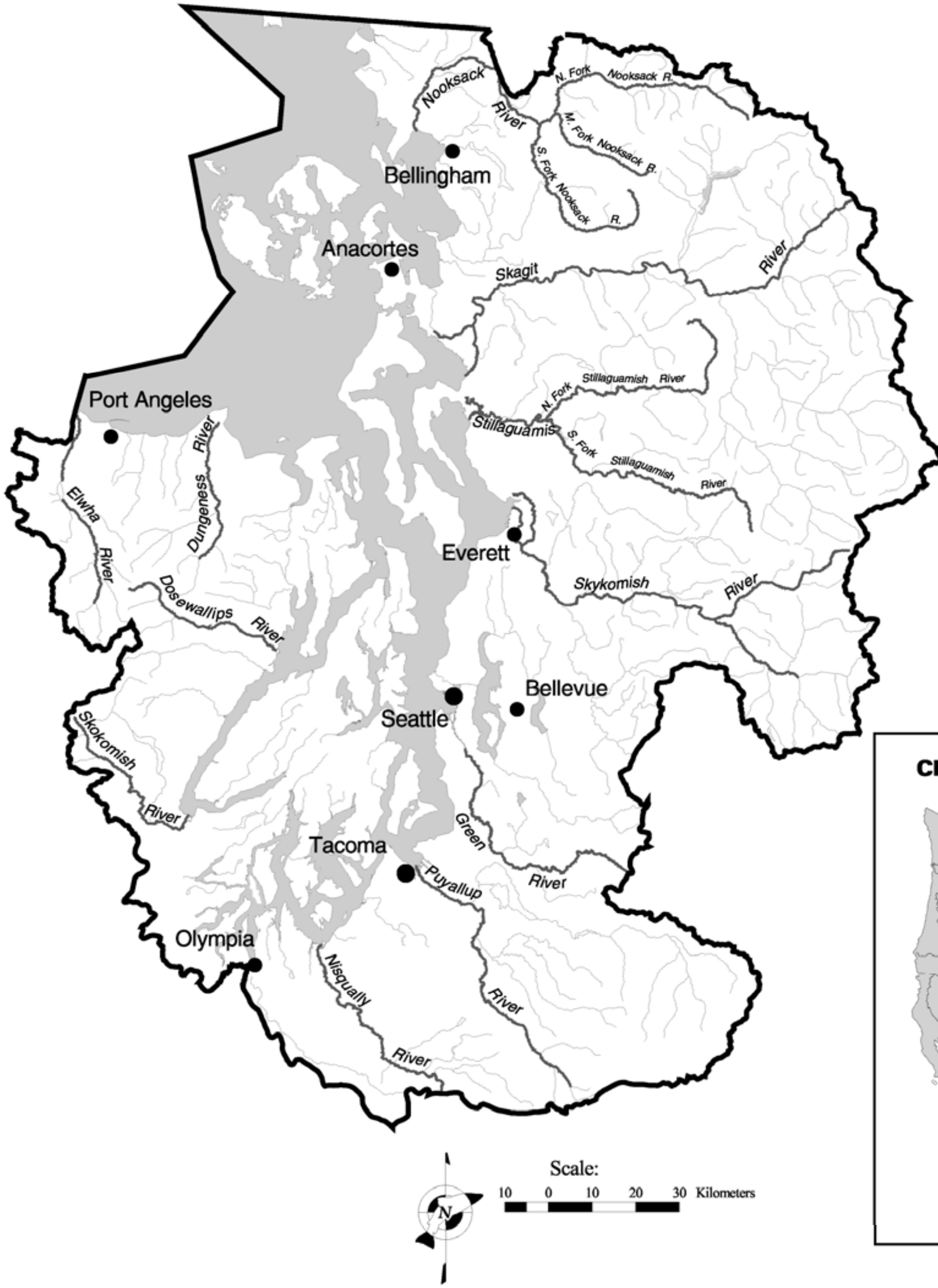
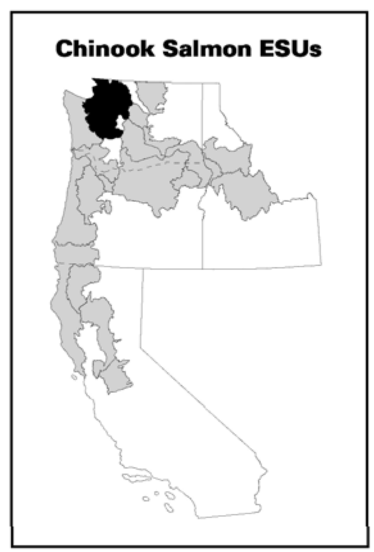
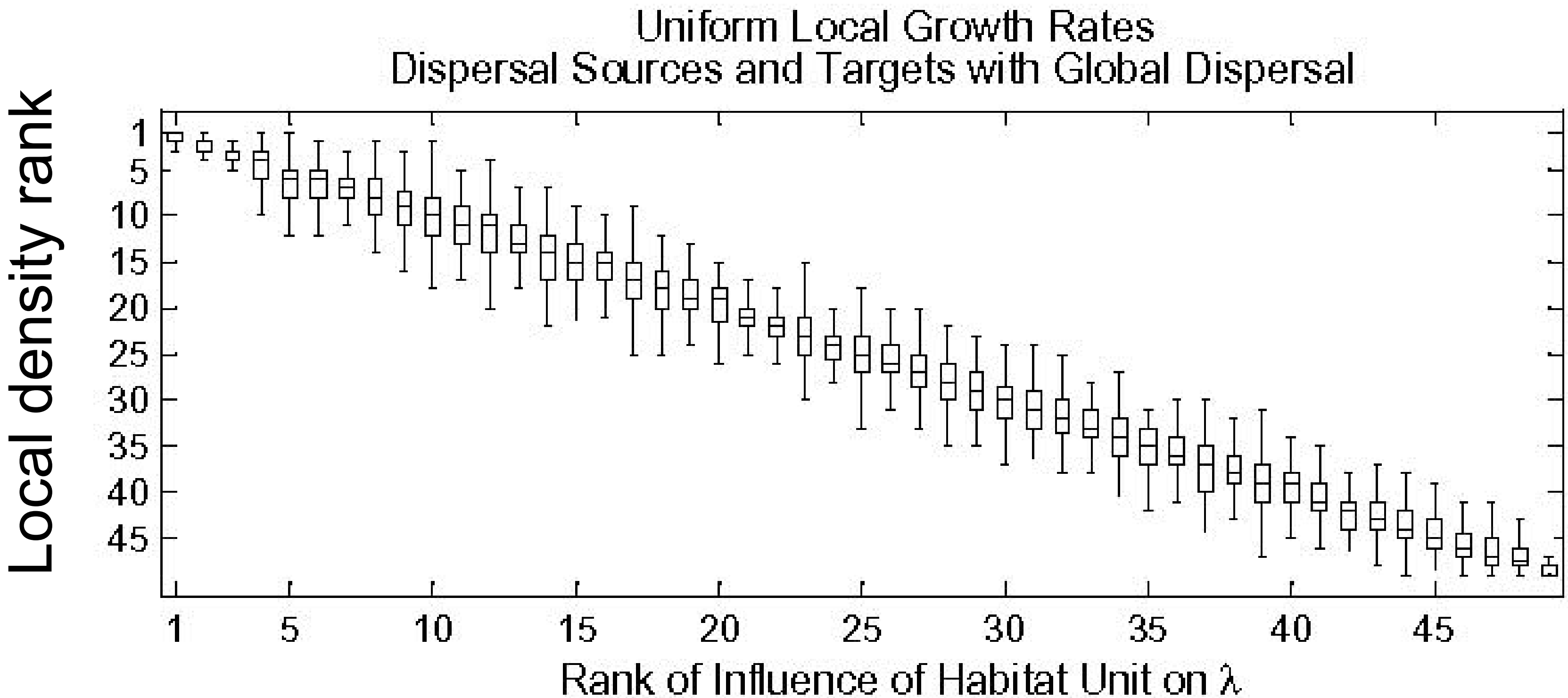
- This has work has implications for how trends of individual stocks in a metapopulation are interpreted.
- How do you account for non-equilibrium dynamics?
- Transients?

Puget Sound Chinook

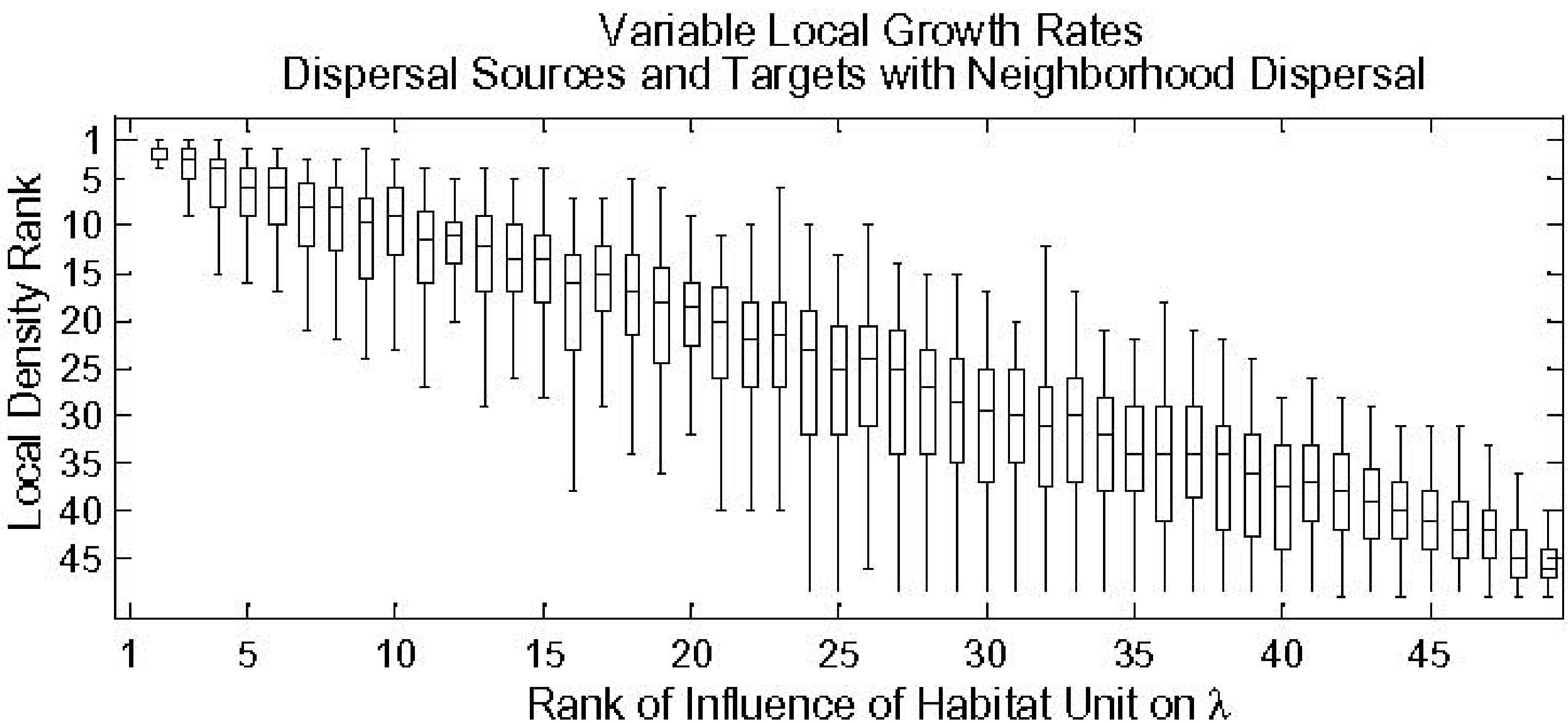




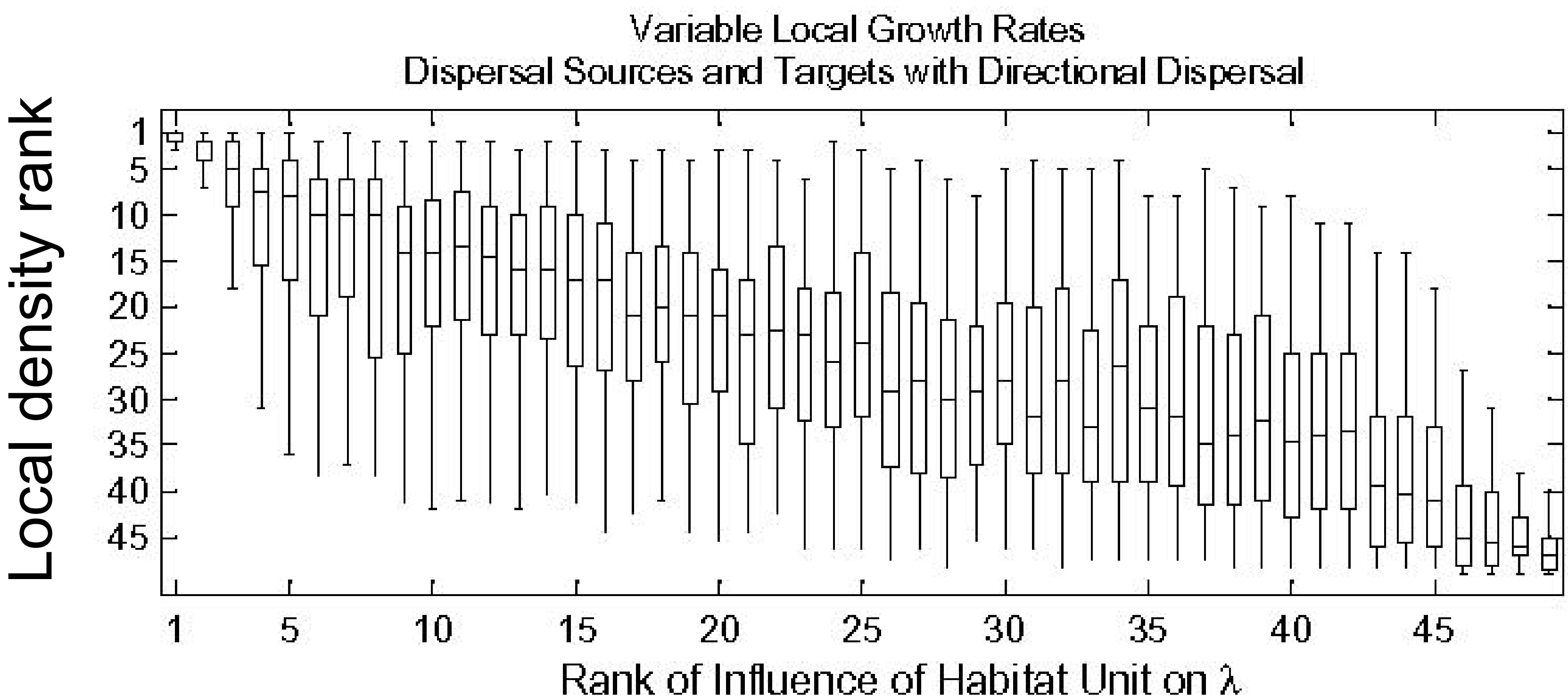
“bang-for-effort”

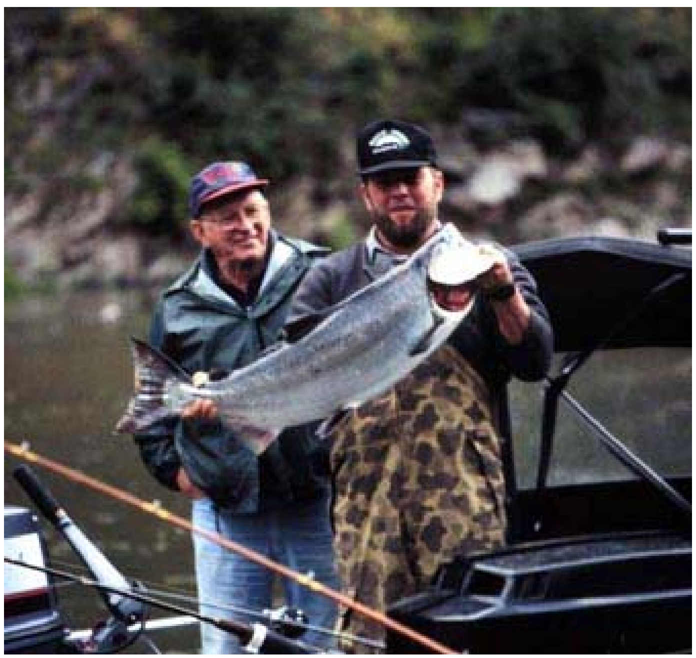


“bang-for-effort”



"bang-for-effort"





- Somewhat intuitive implications for distribution of habitat protection efforts → protect pristine areas with high population densities
- Counter-intuitive in-stream harvest closures → allow harvest in low density-streams and close harvest in high-density streams