Leslie matrices for marine mammals and conservation biology

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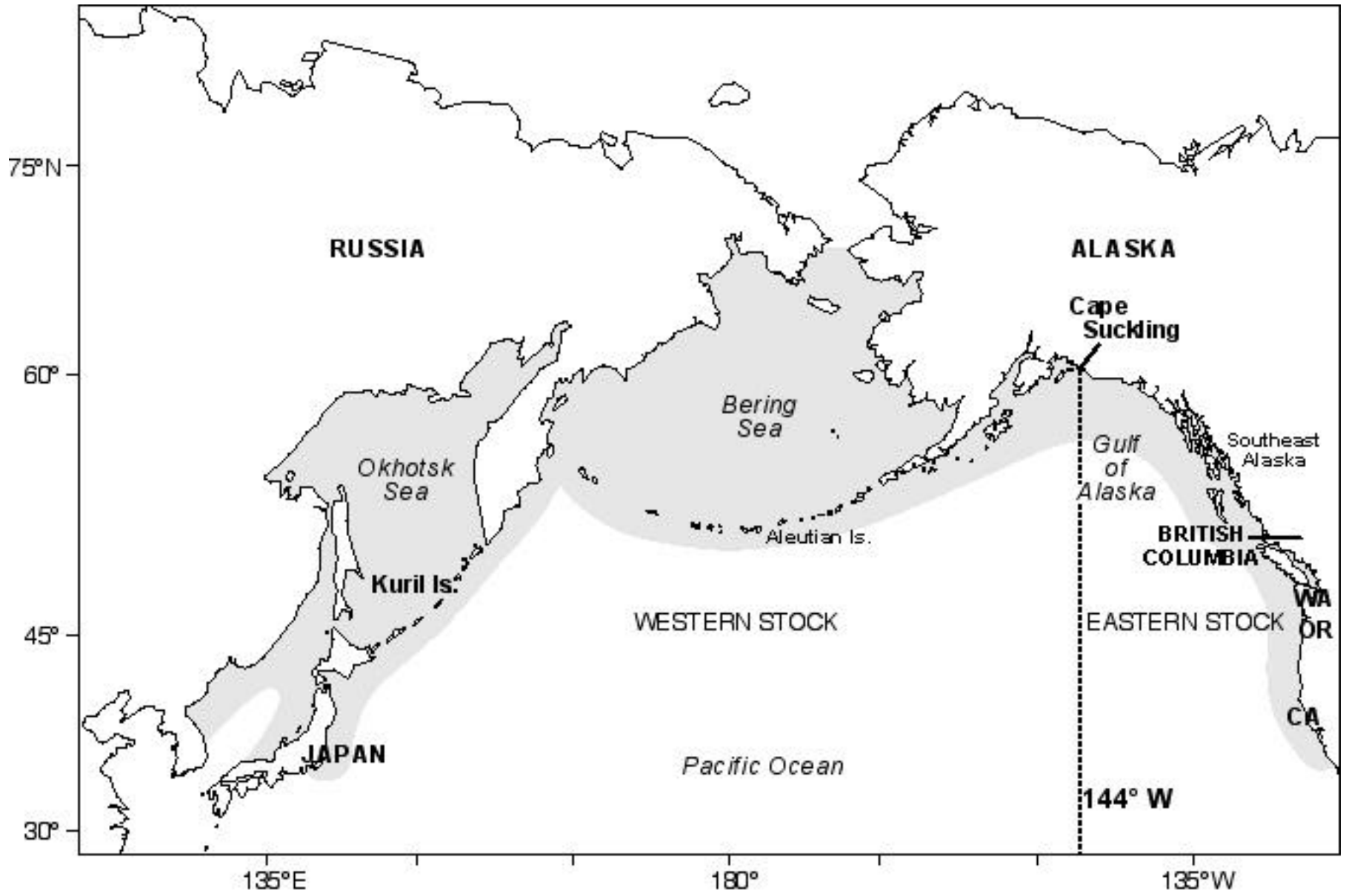
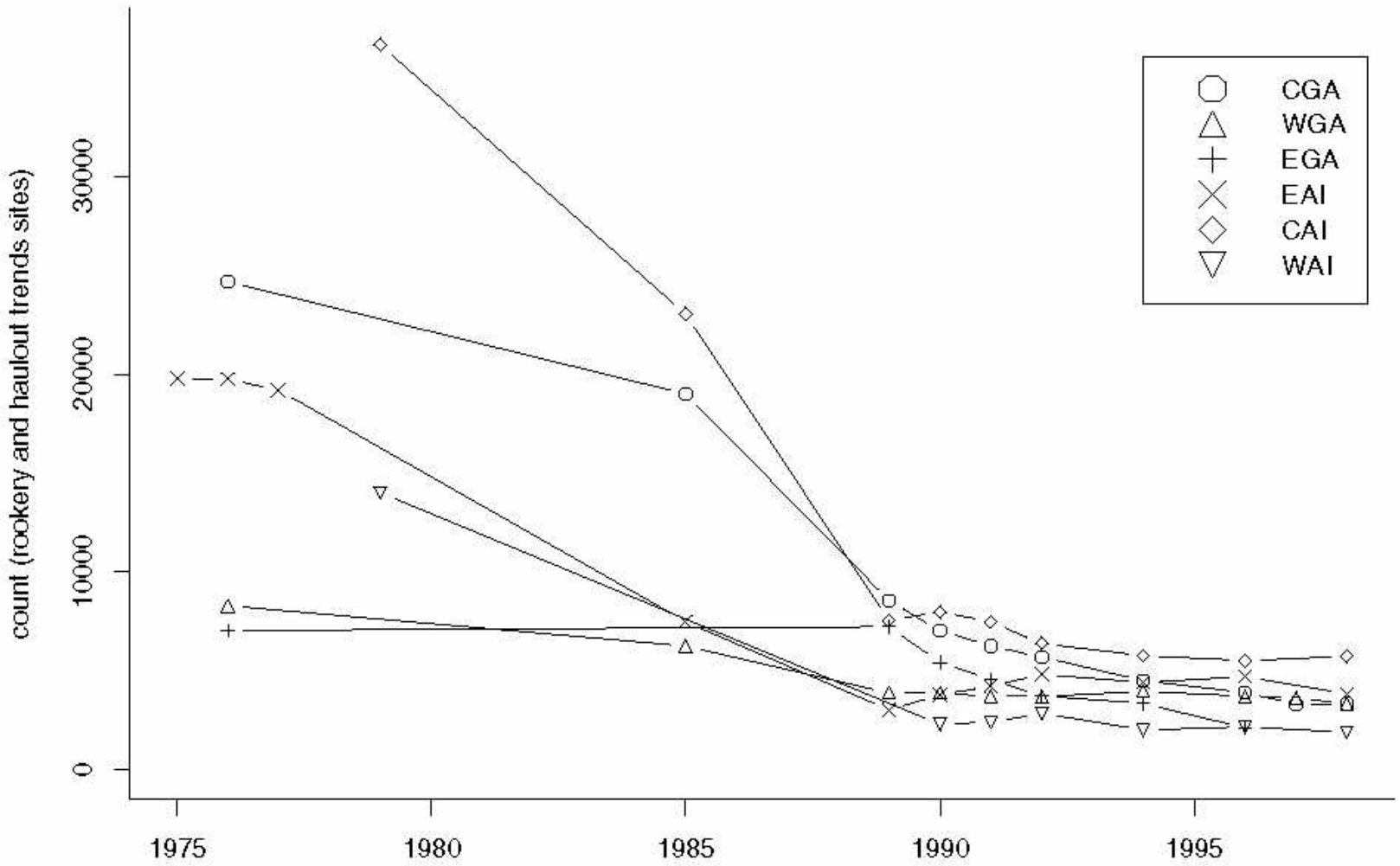

75°N
60°
45°
30°
135°E
180°
135°W
RUSSIA
ALASKA
Cape
Suckling
Bering
Sea
Okhotsk
Sea
Gulf
of
Alaska
Southeast
Alaska
Aleutian Is.
BRITISH
COLUMBIA
Kuril Is.
WESTERN STOCK
EASTERN STOCK
WA
OR
CA
JAPAN
Pacific Ocean
144° W

1969

1979

1986


count (rookery and haulout trends sites)
0
10000
20000
30000
1975
1980
1985
1990
1995
CGA
WGA
EGA
EAI
CAI
WAI

The most common analyses in conservation biology using life-history modeling are

- What is the population rate of decline?
- To what vital rate is the population growth rate most responsive?
- How fast will the population go extinct?
- What happened to the population in the past?

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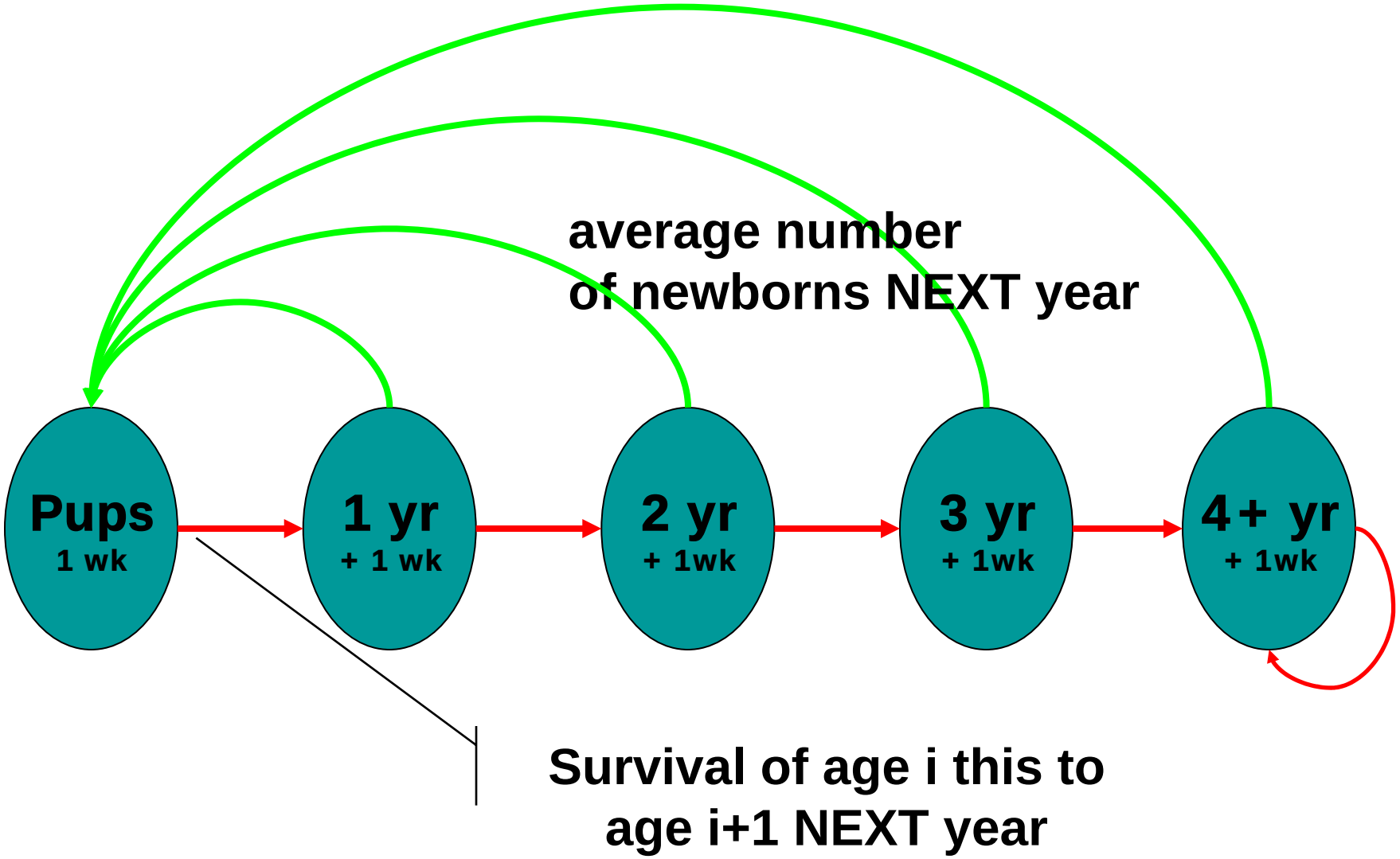
- Life-history models and Leslie matrices
- Most important attributes of Leslie matrix models
  - Long-term rate of increase, λ
  - Stable age-distribution
- How do you estimate a Leslie matrix?
- Two types of common analyses using Leslie matrices
  - Sensitivity analysis
  - Historical analysis

A life-history model translates numbers THIS year to numbers NEXT year



Let’s make this into a generic female pinniped model

- Start reproducing at age 4
- On average 60% of females have a pup every year
- 50:50 sex ratio of newborns
- Lower juvenile survivorship
- High adult survivorship

s1 = .8; s2,3,4 = .9; f3 = .3

	2006	2007	2008
pup	100	60 =200*.3	135
Age 1	800	80 =100*.8	48
Age 2	500	720 =800*.9	72
Age 3	300	450 =500*.9	648
Age 4	200	450 =300*.9+200*.9	810

Write the model as a **Leslie Matrix**

- Translates numbers this year to next year
- Top row is fecundity
- Other rows are survivorship

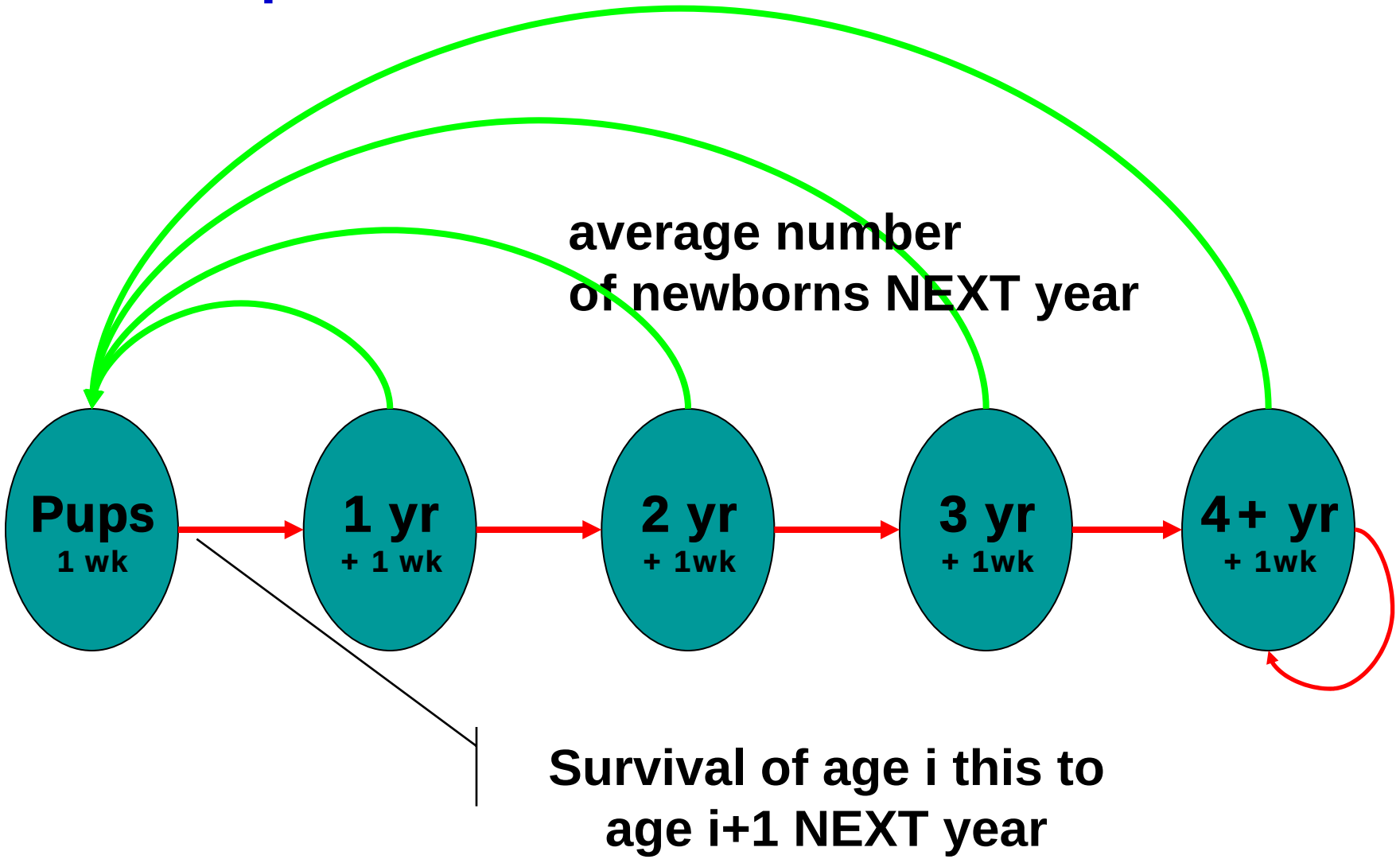
	Pups	Age 1	Age 2	Age 3	Age 4+
Pups	0	0	0	0	.3
Age 1	.8	0	0	0	0
Age 2	0	.9	0	0	0
Age 3	0	0	.9	0	0
Age 4+	0	0	0	.9	.9

Let's project the population forward with matrix algebra

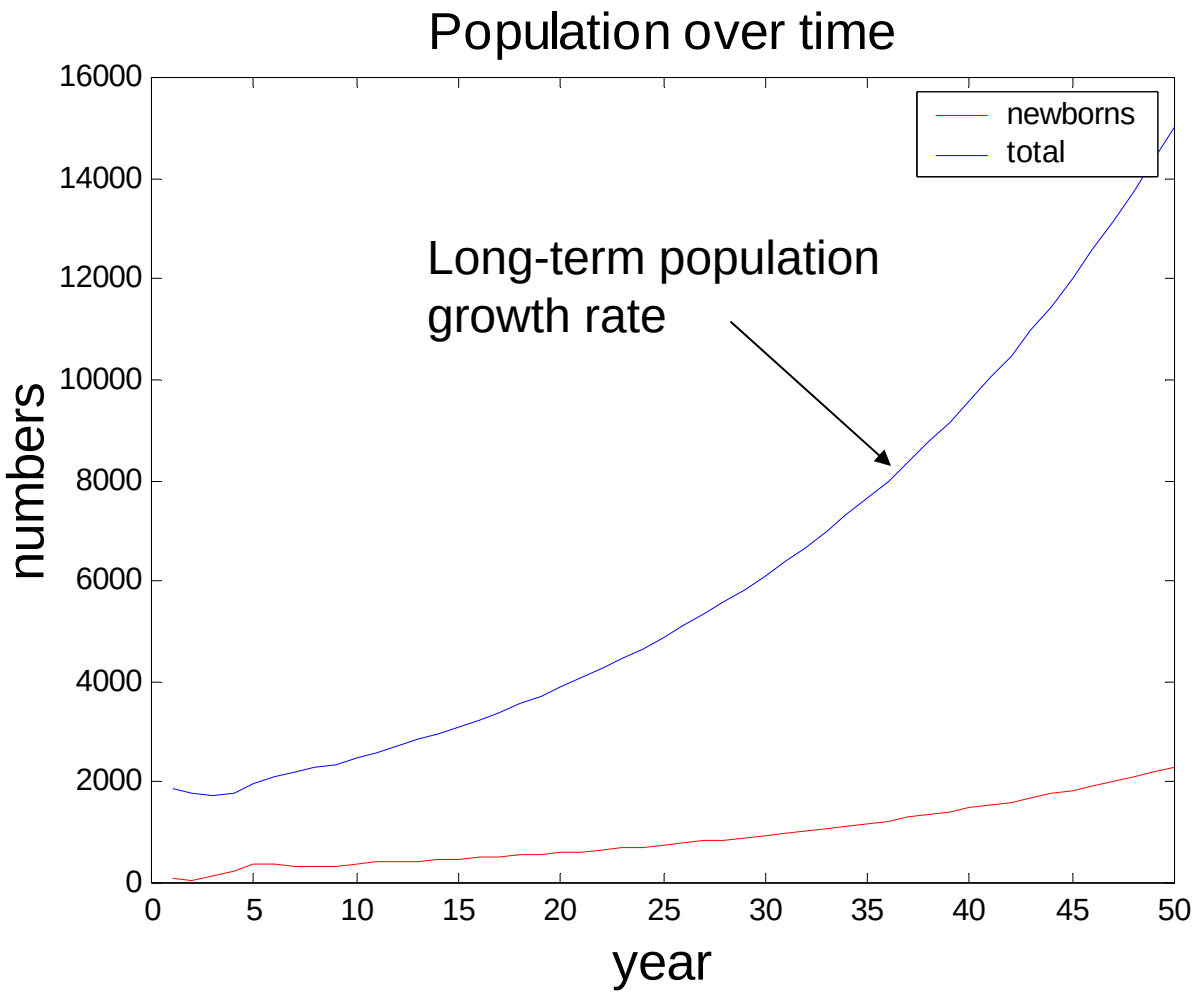
$$\begin{matrix}\text{Pups}\\ \text{Age 1}\\ \text{Age 2}\\ \text{Age 3}\\ \text{Age 4+}\end{matrix}\begin{bmatrix}60\\80\\720\\450\\450\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 0 & .3\\ .8 & 0 & 0 & 0 & 0\\ 0 & .9 & 0 & 0 & 0\\ 0 & 0 & .9 & 0 & 0\\ 0 & 0 & 0 & .9 & .9\end{bmatrix} \times \begin{bmatrix}100\\800\\500\\300\\200\end{bmatrix}$$

Red is pups next year by females age j in this year
Blue is number of age j animals this year that survive to age j+1 next year

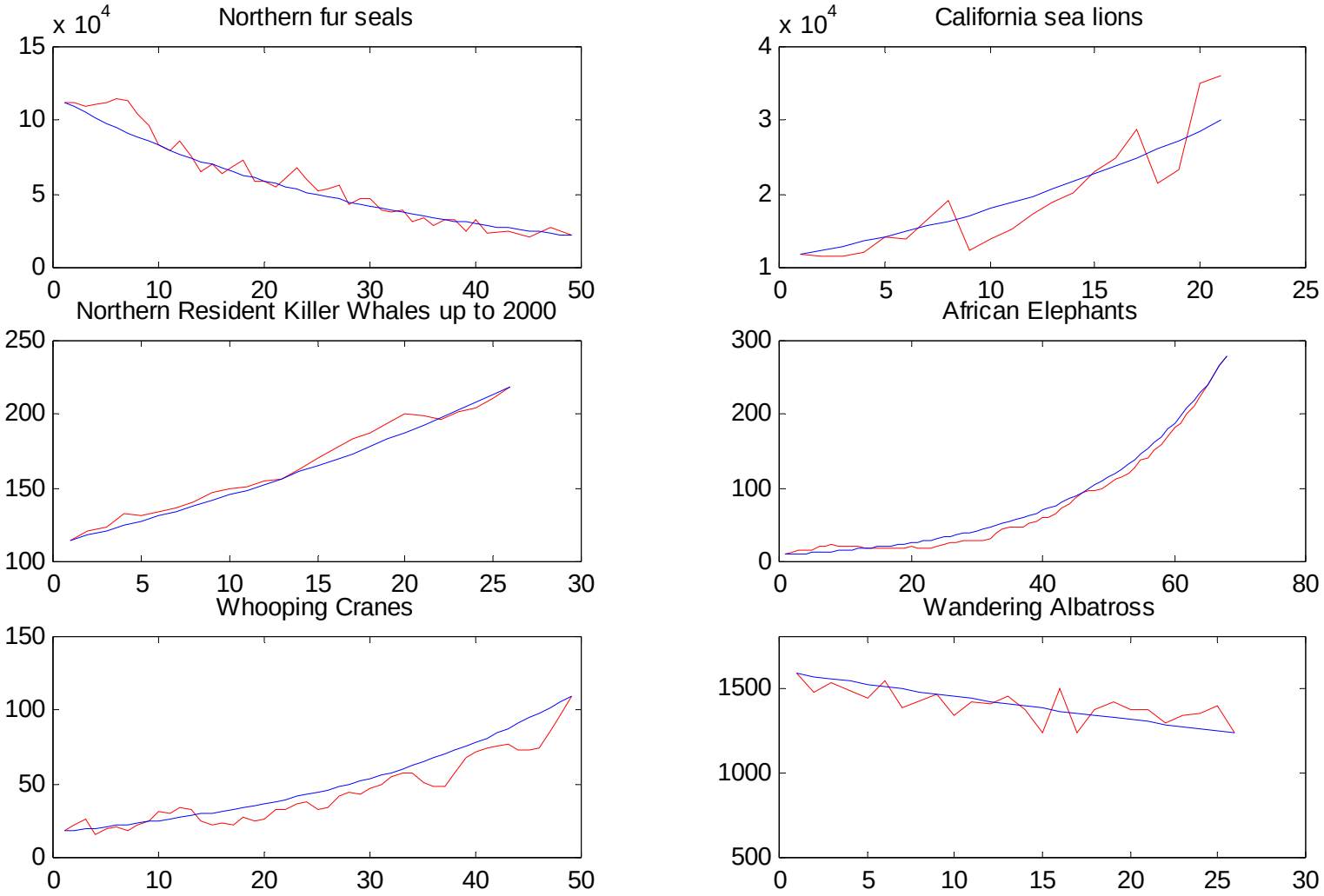
So the life-history cartoon is also a picture of a Leslie matrix



After awhile, the population starts to grow (or decline) exponentially



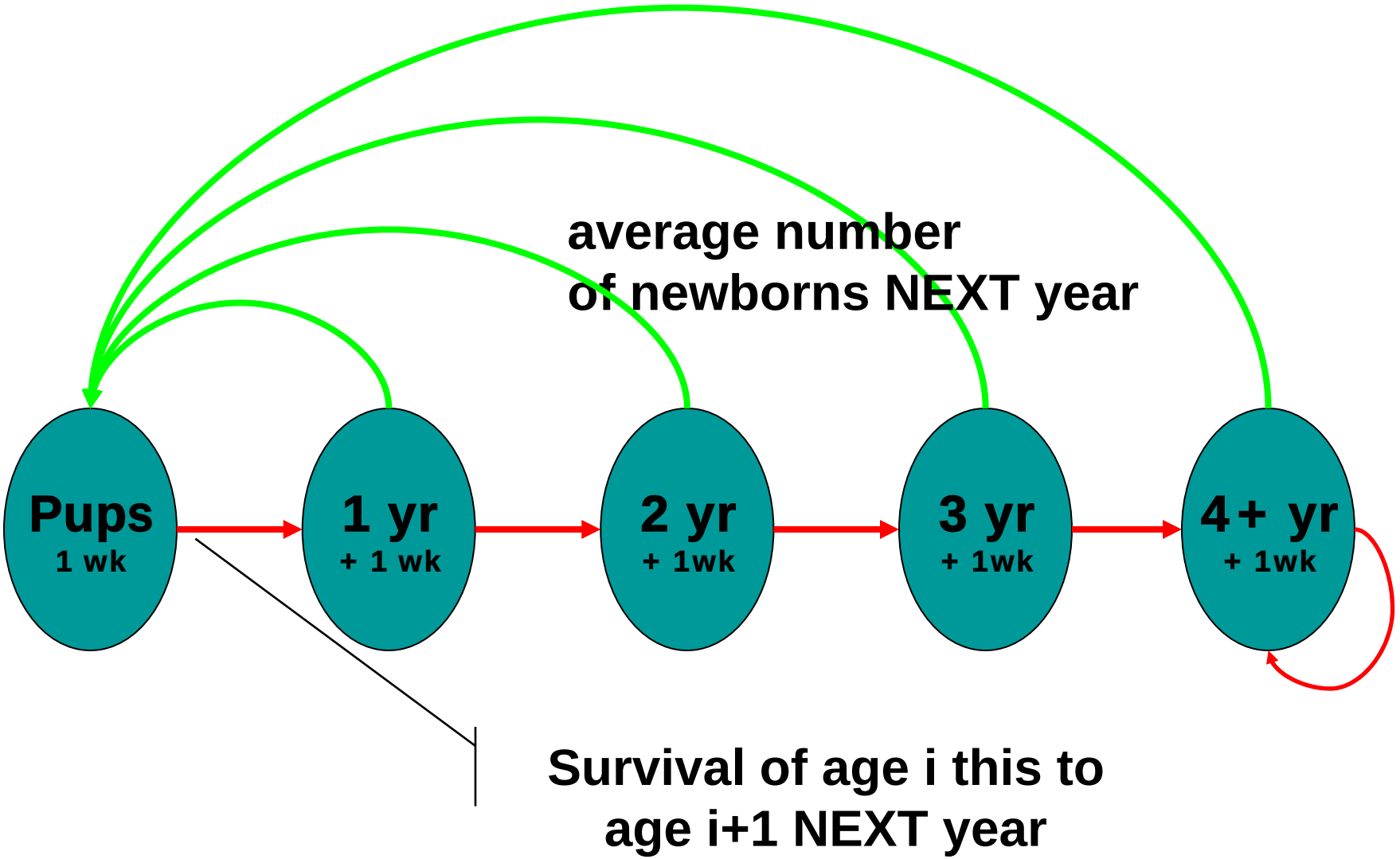
Some real population trajectories for long-lived species



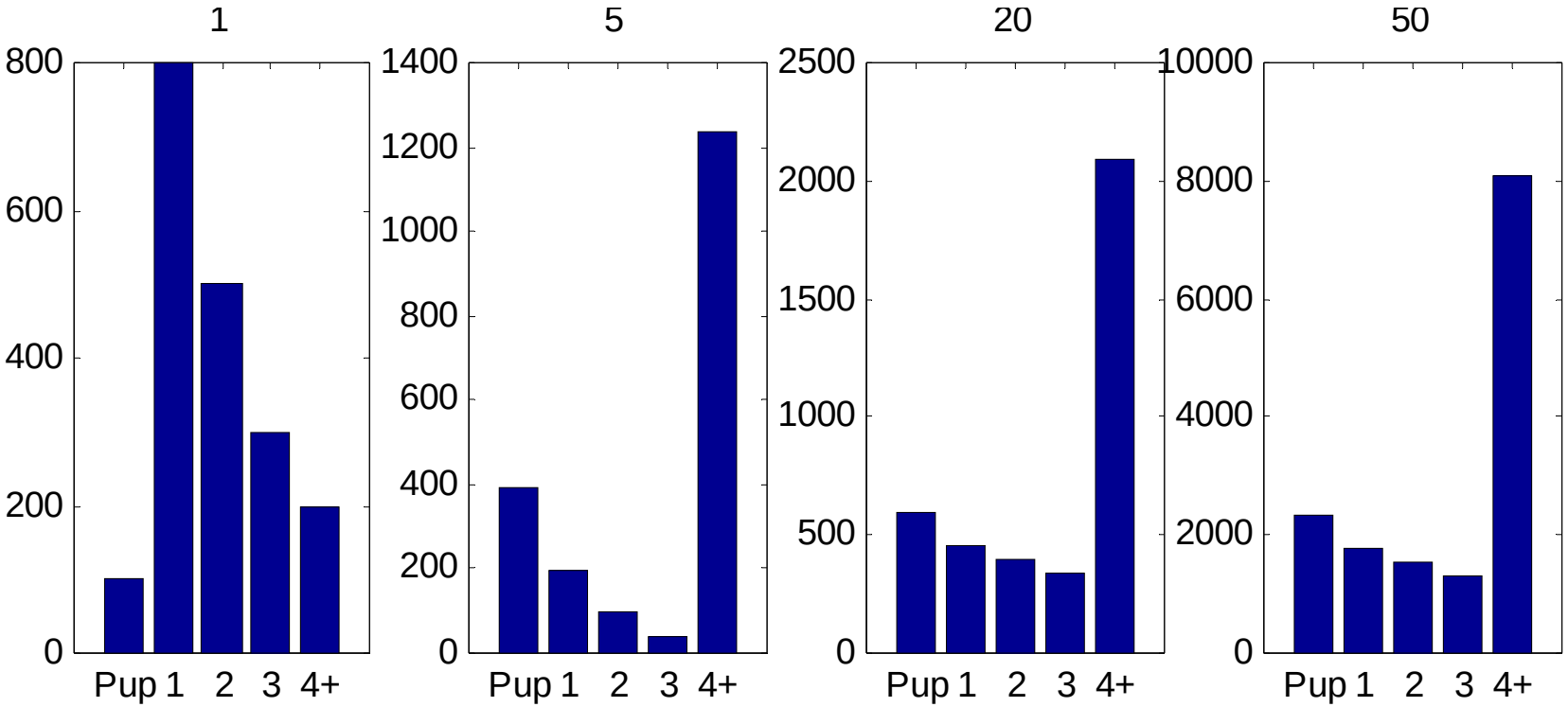
Holmes & Fagan (2006) An analysis of IUCN risk criteria

Long-term population growth rate

- Termed λ
- **The λ = maximum eigenvalue of the Leslie matrix**
- What's an eigenvalue?
- Take-home messages
  - A life-history model goes to a steady exponential rate of increase (or decrease).
  - Max. eigenvalue of the Leslie matrix is the long-term growth rate
  - If I were able to estimate the Leslie matrix, I could easily estimate the long-term rate of increase


average number
of newborns NEXT year
Pups
1 wk
1 yr
+ 1 wk
2 yr
+ 1wk
3 yr
+ 1wk
4+ yr
+ 1wk
Survival of age i this to
age i+1 NEXT year

After a few years the proportion of animals in each age class stabilizes (it doesn't take long)



Long-term population age-distribution

- **The stable age-distribution = dominant eigenvector of the Leslie matrix**
- Take-home messages
  - A life-history model goes to a stable age-distribution.
  - If I were able to estimate the Leslie matrix, I could easily estimate the long-term stable age-distribution.
  - I could compare my population's actual age-distribution to the stable one.

Leslie matrices, marine mammals and conservation biology

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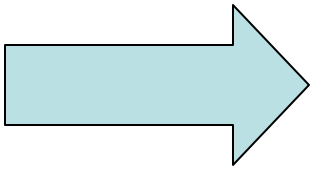
The Leslie matrix has 2 parts: fecundity and survival

Leslie matrix model

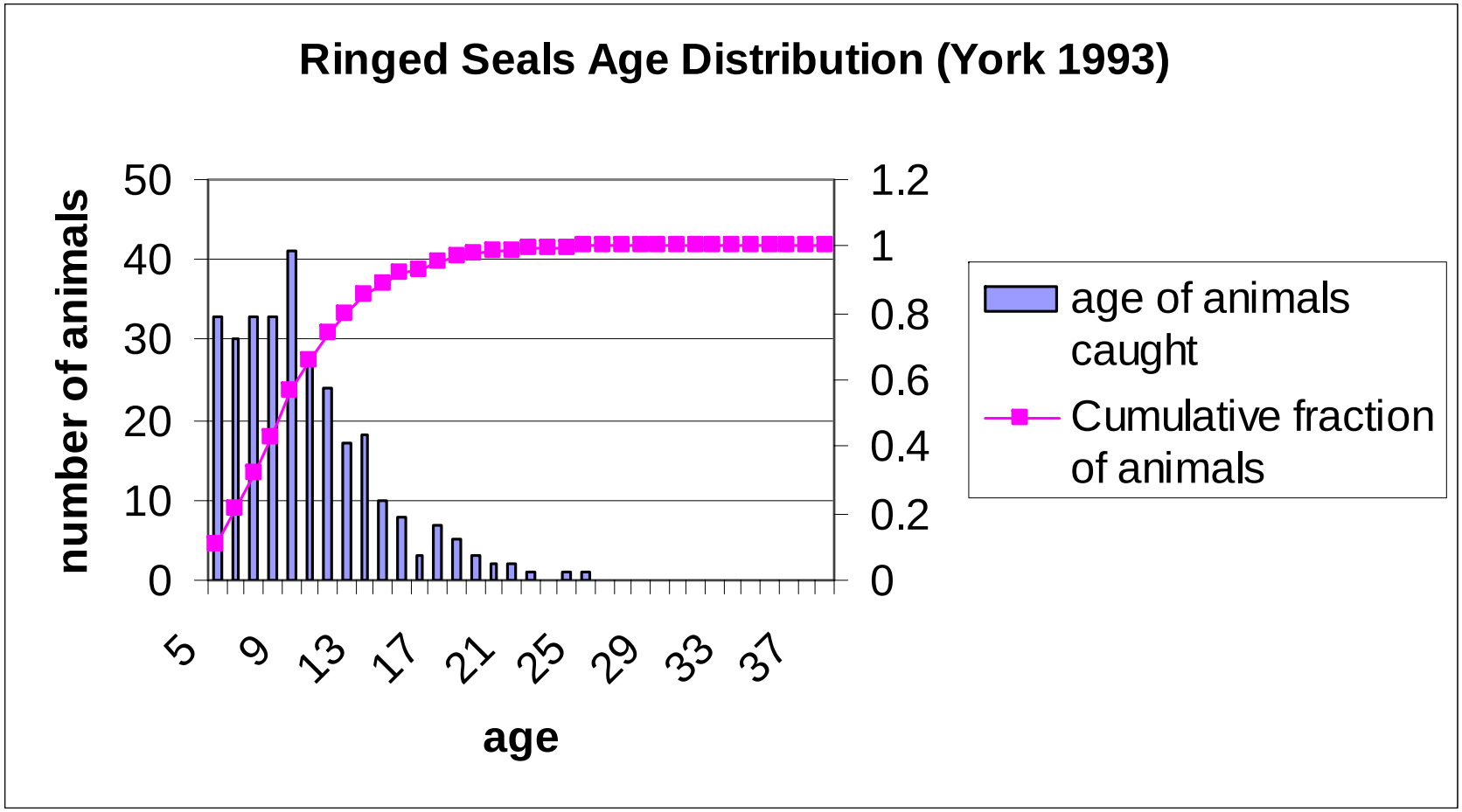


$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1.2 \\ .8 & 0 & 0 & 0 & 0 \\ 0 & .9 & 0 & 0 & 0 \\ 0 & 0 & .9 & 0 & 0 \\ 0 & 0 & 0 & .9 & 0 \end{bmatrix}$$

Estimating survival from mark-capture-recapture studies

- Capture and mark individuals or id them (photo-id typically)
- Sight them (not actually recapture them) in subsequent years
- Use a **Cormack-Jolly-Seber Model** to analyze the data using some program like MARK or SURGE.

Estimating survival from age-structure



Where do reproductive estimates come from?

- Long-term cohort studies – basically long-term field studies that follow individuals using marks or photo-id
- Opportunistic analysis of dead animals (examine the uterus) or analysis of a deliberate large sample of dead animals

Average number of newborns born next year

X fraction of females at age i that are mature

X fraction of mature females that are impregnated

X fraction of impregnated females that survive to the next year

X fraction of early term pregnancies that make it to late-term and birth

X fraction of newborns that survive to census date

= average number of newborns born NEXT year to females age i THIS year

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Sensitivities (and elasticities)

Definition

- What is the relative change in λ when specific vital rates are increased by a small amount (sensitivity)?
- What is the relative change in λ when specific vital rates are changed by a small percentage (elasticity)?

How much does λ change when one (or a combo) of the elements is changed?

- Sensitivity: A_{ij} + a tiny amount
- Elasticity: A_{ij} X a tiny percentage

$$\begin{bmatrix} 0 & 0 & 0 & 0 & .3 \\ .8 & 0 & 0 & 0 & 0 \\ 0 & .9 & 0 & 0 & 0 \\ 0 & 0 & .9 & 0 & 0 \\ 0 & 0 & 0 & .9 & .9 \end{bmatrix}$$

- You calculate sensitivities or elasticities directly from the Leslie matrix

A famous example of elasticities affecting management of a long-live species

A STAGE-BASED POPULATION MODEL FOR LOGGERHEAD SEA TURTLES AND IMPLICATIONS FOR CONSERVATION[1]

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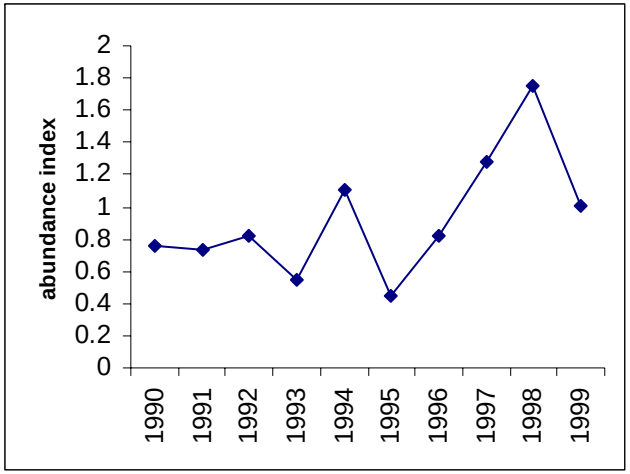
Abstract. Management of many species is currently based on an inadequate understanding of their population dynamics. Lack of age-specific demographic information, particularly for long-lived iteroparous species, has impeded development of useful models. We use a Lefkovitch stage class matrix model, based on a preliminary life table developed by Frazer (1983*a*), to point to interim management measures and to identify those data most critical to refining our knowledge about the population dynamics of threatened loggerhead sea turtles (*Caretta caretta*). Population projections are used to examine the sensitivity of Frazer's life table to variations in parameter estimates as well as the likely response of the population to various management alternatives. Current management practices appear to be focused on the least responsive life stage, eggs on nesting-beaches. Alternative protection efforts for juvenile loggerheads, such as using turtle excluder devices (TEDs), may be far more effective.

Key words: Caretta caretta; *demography; endangered species; management; (marine) turtles; stage class matrix projection models; southeastern United States.*



Leslie matrices, marine mammals and conservation biology

- Leslie matrix
- How to convert a cartoon of a life-history model to a Leslie matrix
- What's λ?
- What's a stable age-structure
- What are sensitivities? And elasticities?
- Why would you want to estimate a Leslie matrix for your population of interest?

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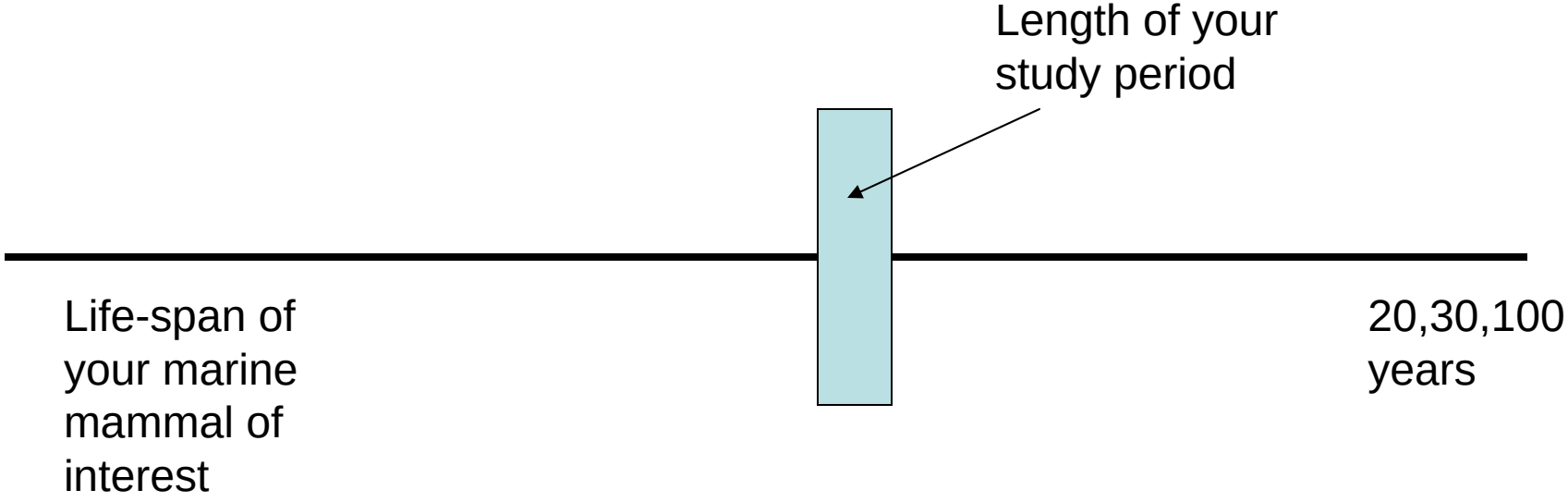
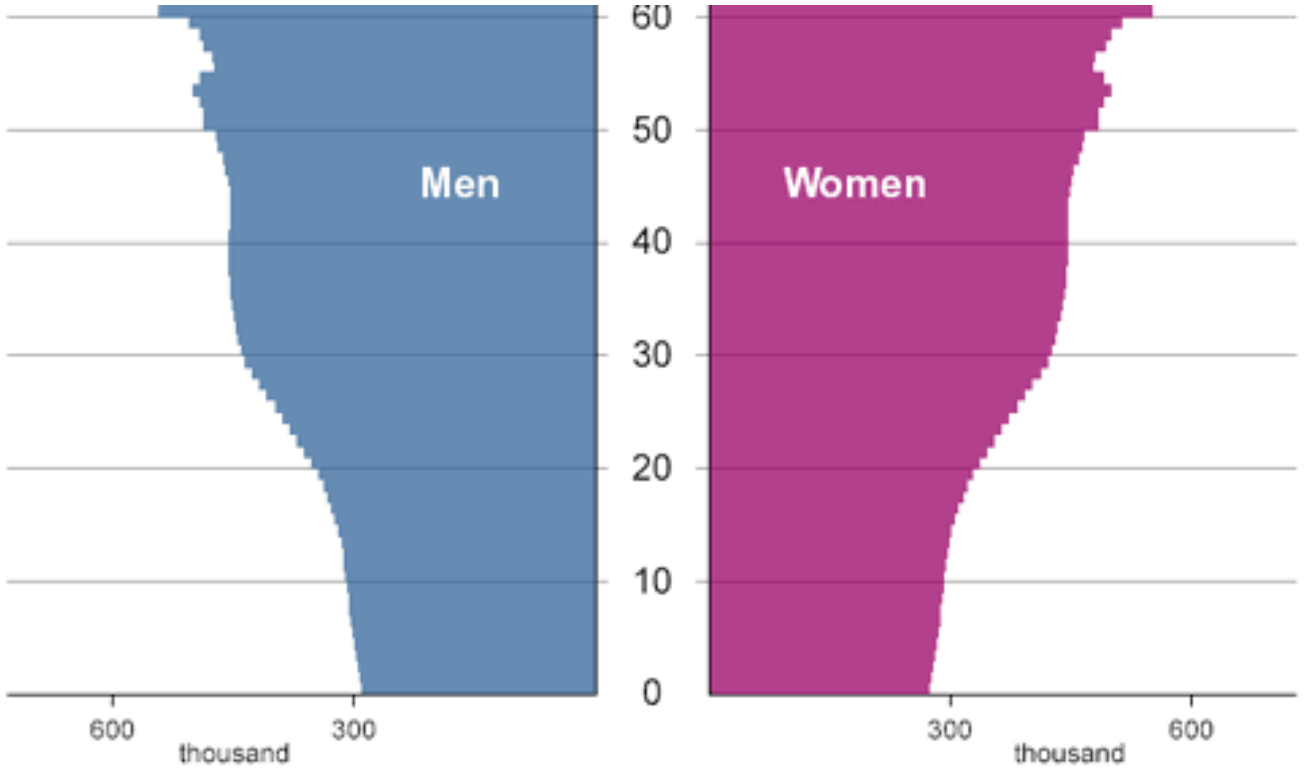
Analyzing impacts from the past in a long-lived species

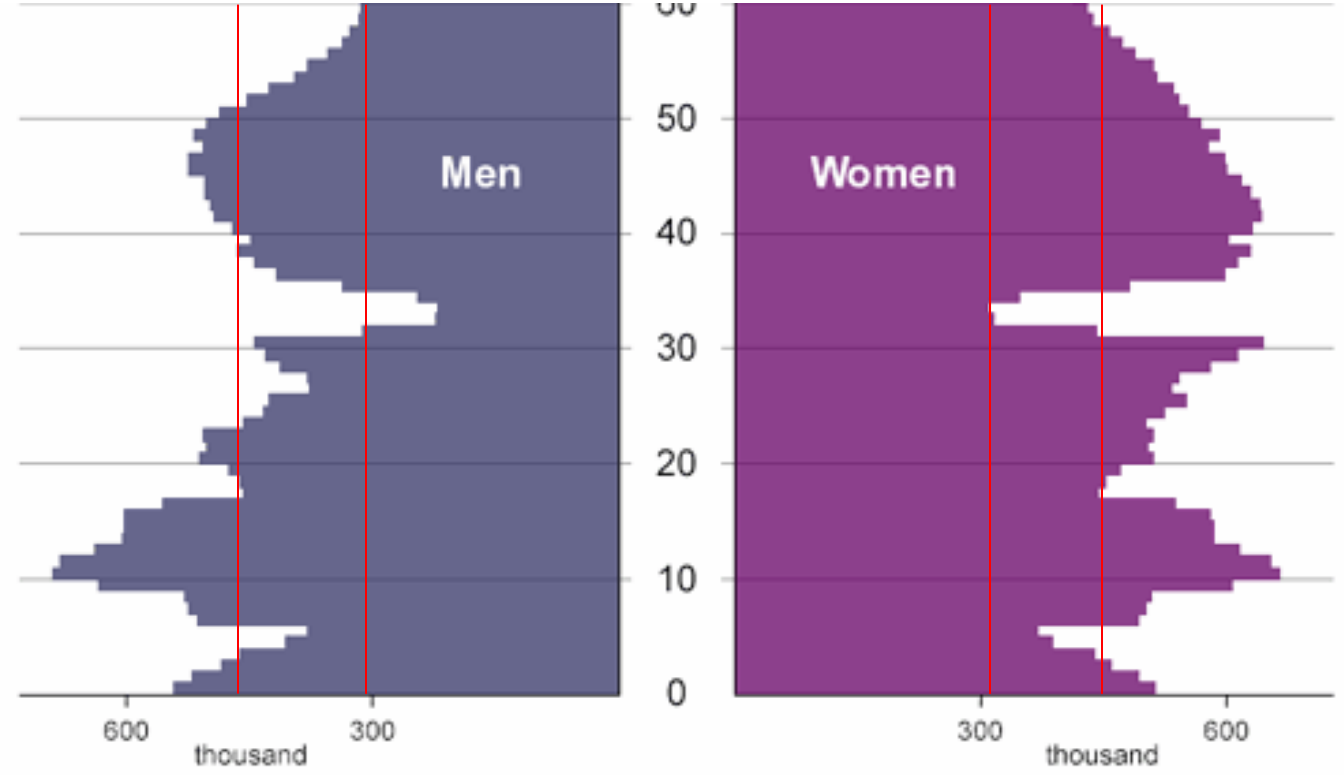


Predicted age-distribution in 2050 in Germany



Federal Statistical Office Germany

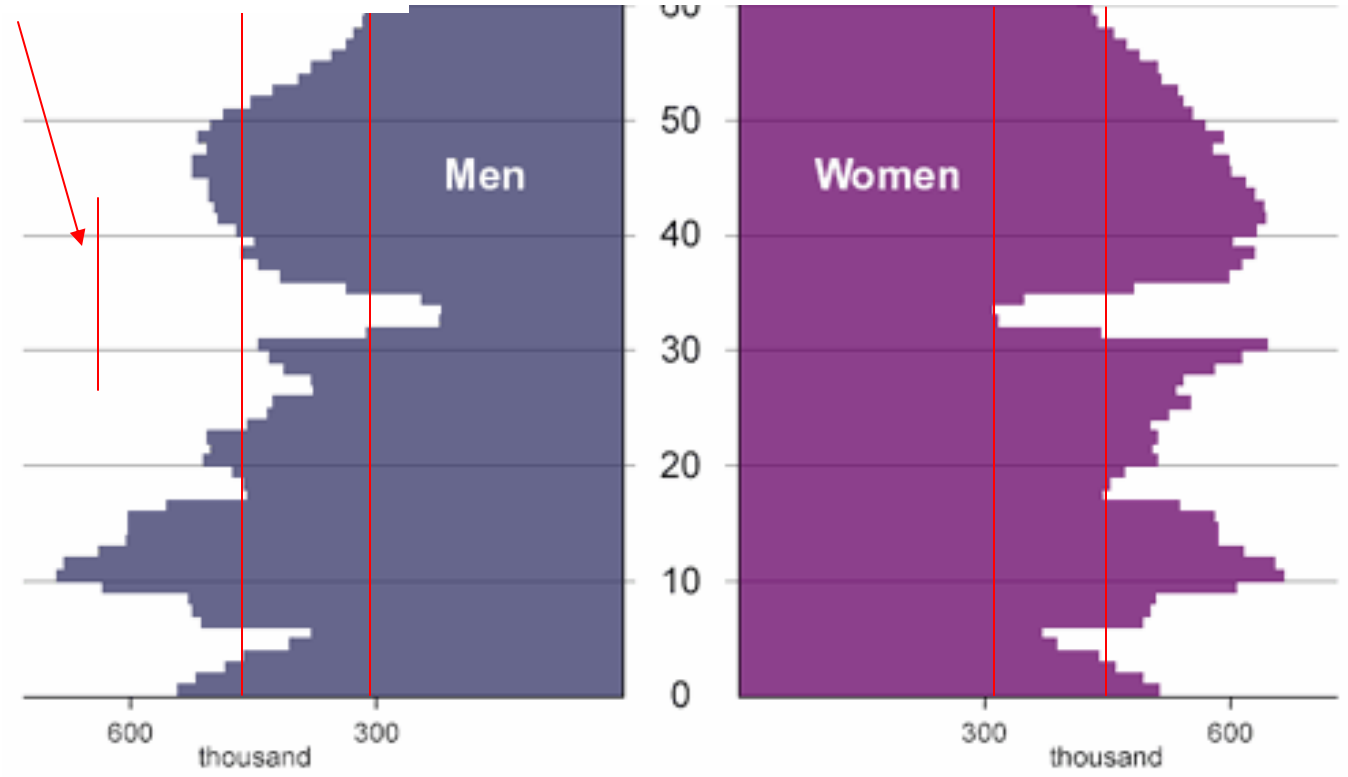
Actual age-distribution in 1950 in Germany



Federal Statistical Office Germany

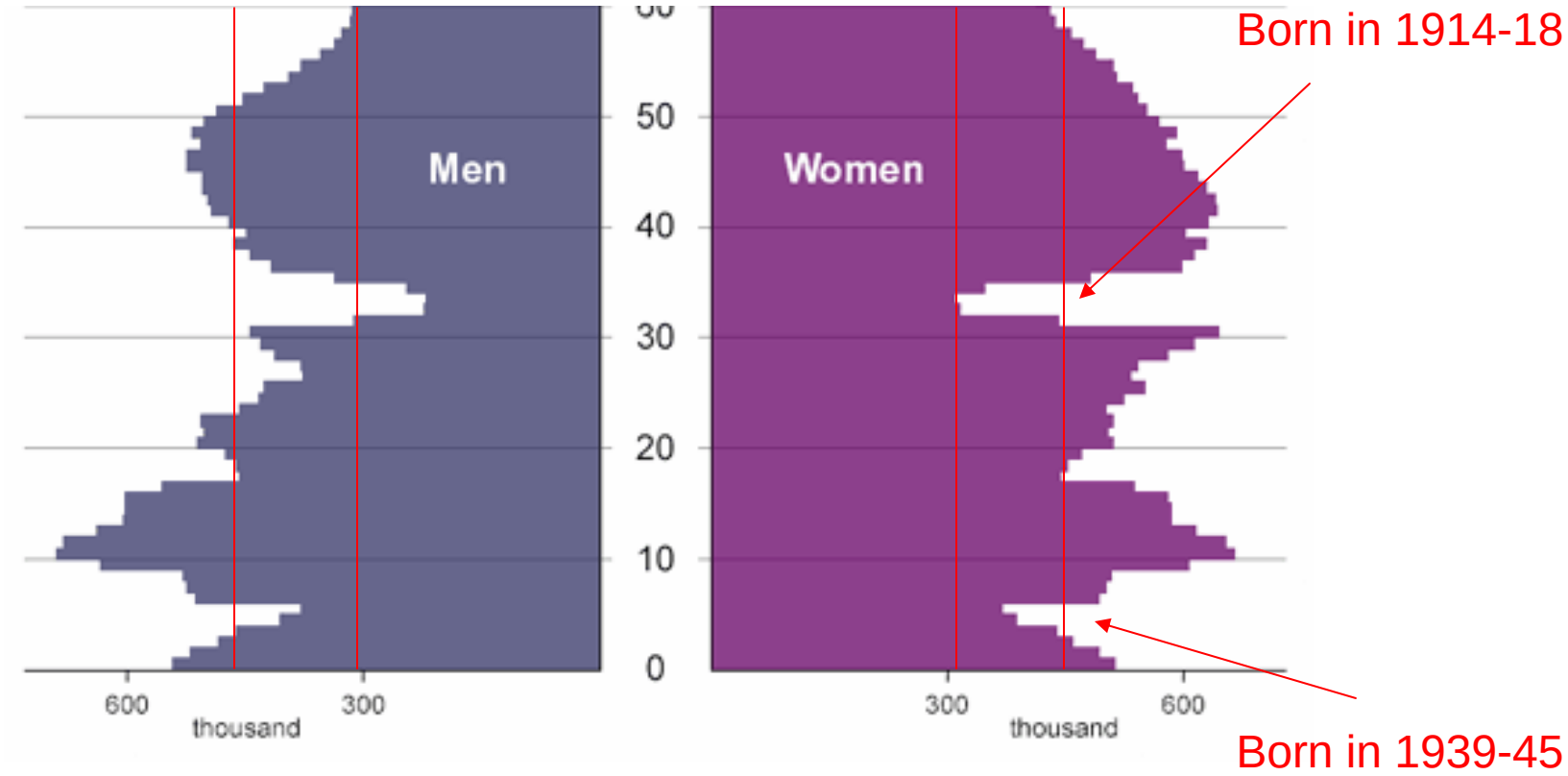
30% of drafted men were killed in WWII

Draft age (18-30) in 1939-1945



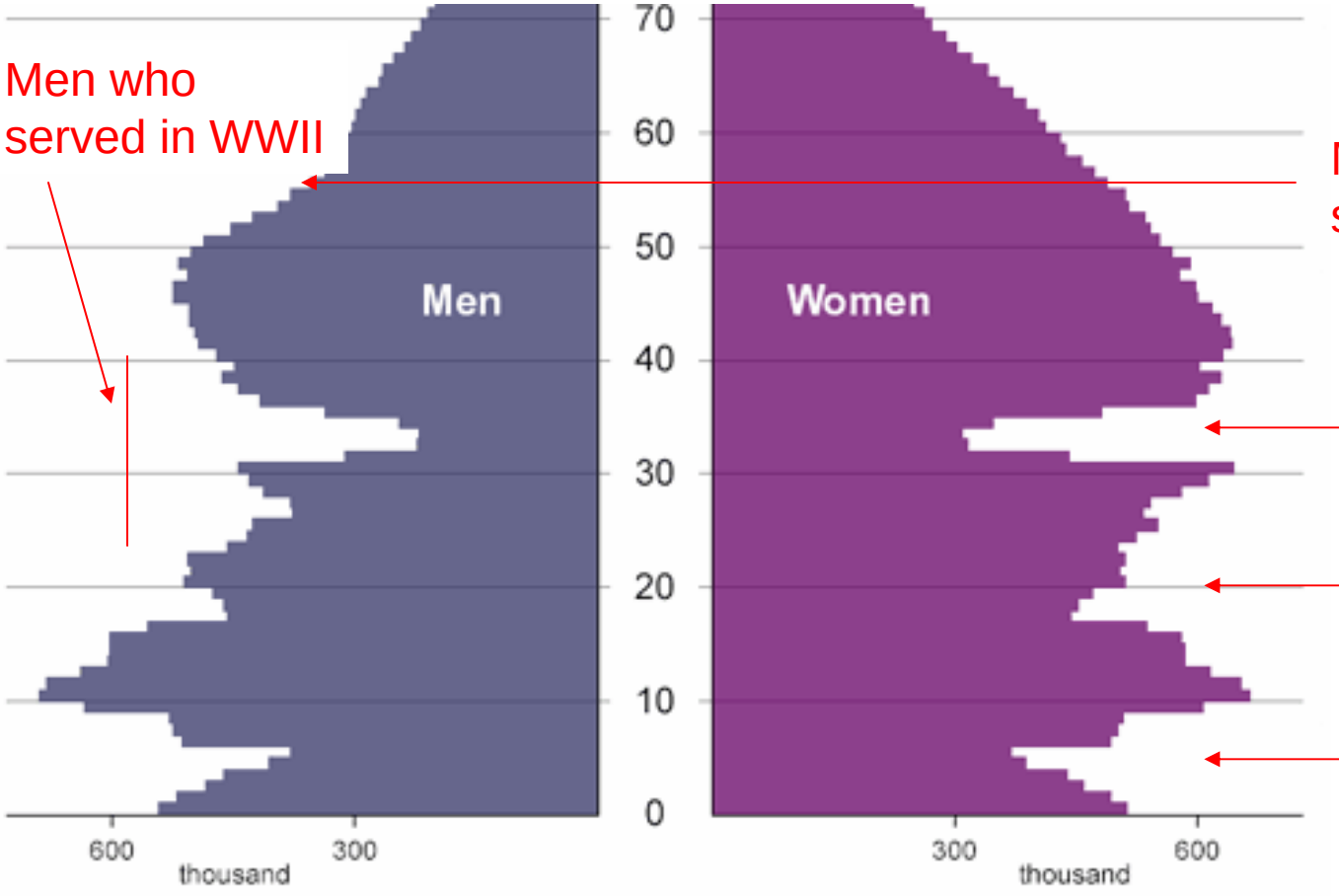
Federal Statistical Office Germany

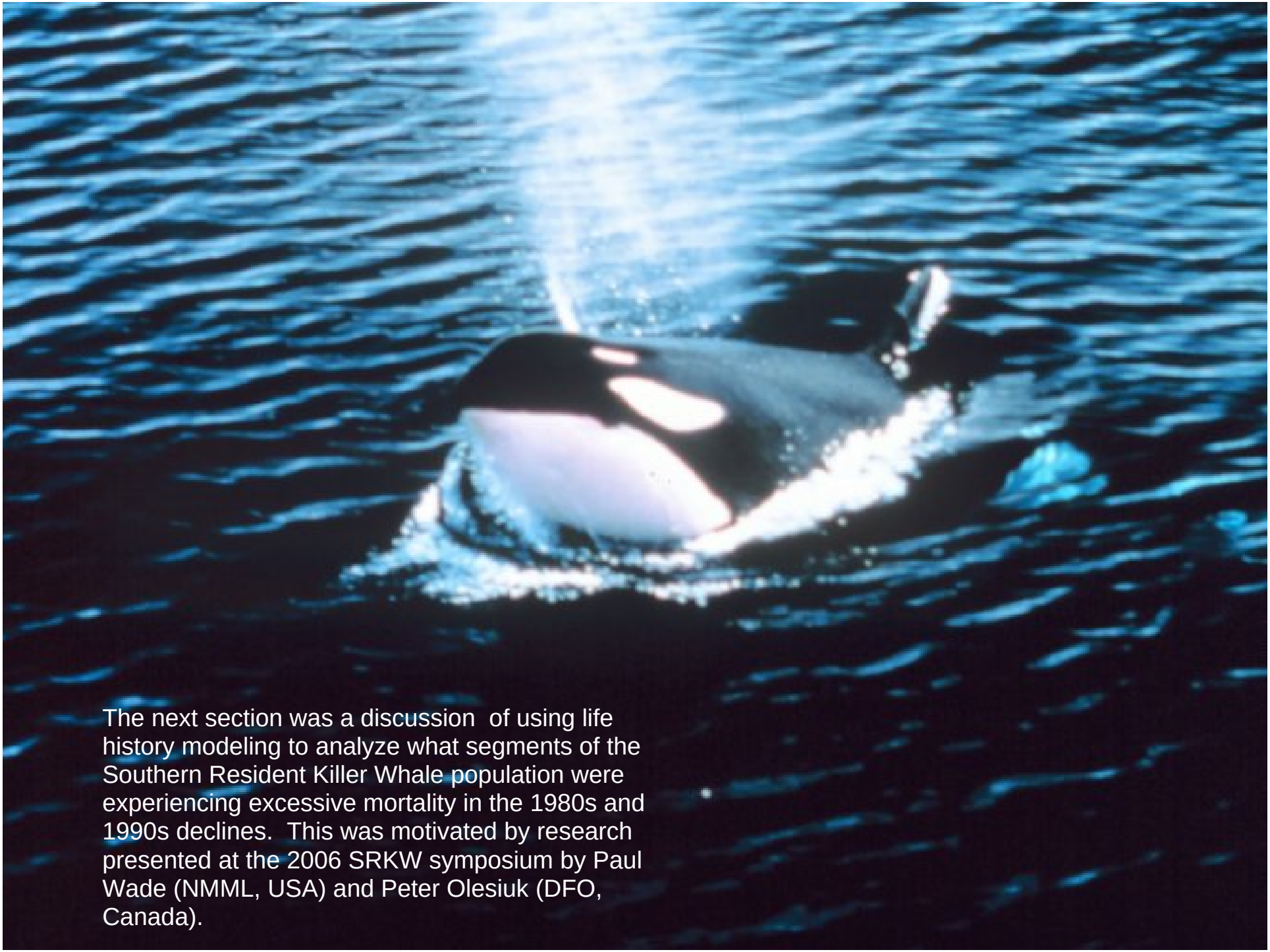
Families don’t have kids during wartime



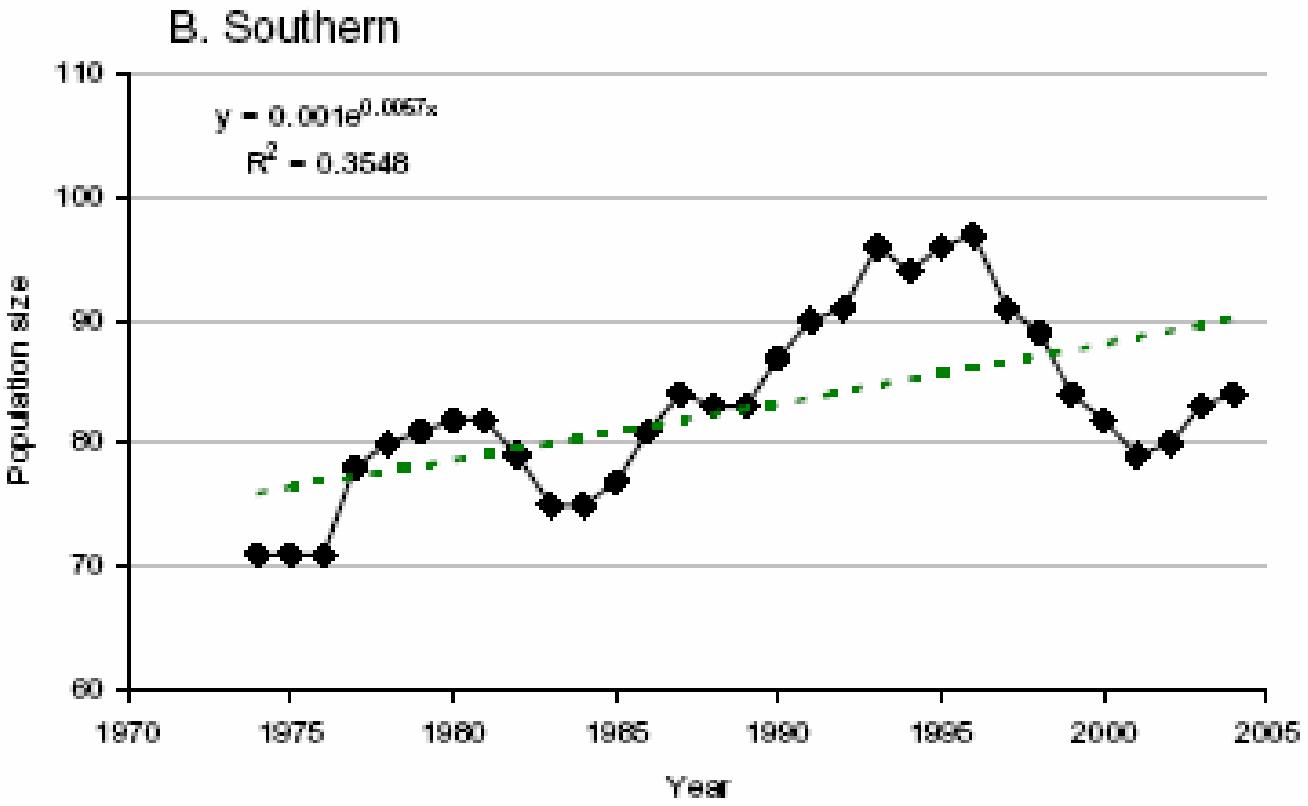
Federal Statistical Office Germany

The age-structure of a long-lived animal is a record of the history the population has experienced.





The next section was a discussion of using life history modeling to analyze what segments of the Southern Resident Killer Whale population were experiencing excessive mortality in the 1980s and 1990s declines. This was motivated by research presented at the 2006 SRKW symposium by Paul Wade (NMML, USA) and Peter Olesiuk (DFO, Canada).


B. Southern
y = 0.001e^0.0057x
R^2 = 0.3548
110
100
90
80
70
60
Population size
1970
1975
1980
1985
1990
1995
2000
2005
Year